

# SANT NIRANKARI PUBLIC SCHOOL , FARIDABAD

Class - XII

Assignment<sub>s</sub>

Subject: Economics

Date

17.3.20-31.3.20

## Chapter 1: INDIAN ECONOMY AT THE TIME OF INDEPENDENCE

**Meaning of Economy:** An economy is a system ( or structure) in which various economic activities related to agriculture, industries, insurance, banking etc are performed.

- These economic activities make production of goods and services on one hand and generate employment opportunities and livelihood on the other hand

### **Economy before the advent of British rule**

- (1) India had an independent , self reliant and prosperous economy before the advent of british rule.
- (2) The Indian economy was known as agricultural economy.
- (3) India was also known for its developed handicraft industries.

### **State of Indian Economy on the Eve of Independence**

- (1) Colonial Economy : The Britishers had the control over the political and economic life in India , which resulted in huge drain of wealth from India.
- (2) Stagnant economy: Level of per capita income remained stable, so it is stagnant economy.
- (3) Backward economy: Indian economy was backward and underdeveloped due to many reasons like low level of productivity , traditional methods of agriculture etc.

### **Occupational structure:**

It refers to the distribution of workforce according to different occupations .i.e agriculture, industry and trade etc.

### **Commercialisation of agriculture:**

Commercialization of agriculture implies production of crops for sale in the market rather than for self consumption.

- Farmers were forced to shift from food crops to commercial food crops like cotton and jute to ensure the supply of raw material for British industries.



## **Subsistence agriculture:**

It is form of farming in which the crops raised are intended to provide for the basic needs of the family with little surplus for marketing.

### **The condition of agriculture sector at the time of Independence**

#### **: 1) Low level of agricultural productivity :-**

Agricultural productivity became very low and this stagnation in agriculture sector was mainly due to systems of land settlement that were introduced by the British Government. The Zamindari system, the profit accruing out of the agriculture sector went to Zamindaris instead of the cultivators. This led to discouragement amongst the cultivators to produce less.

#### **2. High dependence on Monsoon :-**

Agriculture sector was mainly dependent on monsoon. No effort was ever made under the British rule to develop permanent means of irrigation.

#### **3. Lack of Proper Input:-**

Low level of technology, lack of irrigation facility and negligible use of fertilizers, added to aggravate the plight of the farmers and contributed to the dismal level of agricultural productivity.

### **The condition of Industrial sector at the time of Independence**

#### **- 1) Discriminatory Tariff Policy:-**

The British Government allowed tariff free export of raw materials from India and tariff free import of British industrial products into India. But a heavy duty on the export of Indian handicrafts products. It leads to decay of handicrafts industry in India.

#### **(ii) Competition from machine:-**

Industrial revolution in Britain gave a stiff competition to the handicraft industries in India. Due to low cost and better quality product produced by machine forced the Indian craftsmen to shut down the handicraft Industry in India.

#### **(iii) New Patterns of Demand:-**

Owing to British rule in India, a new class of people emerged in India. This changed the pattern of demand in India against the Indian products and in favour of British products. As a result, the Indian Industry tended to Perish

#### **(iv) More market for British Goods:-**

An introduction of railways facilitated the transportation of the British products to different parts of the country. As a result, the size of the market for the low cost British product expanded while it started shrinking for the high cost Indian products. This led to decay of Industry in India.

### **The condition of foreign trade under the British rule**

Ans 1) Due to discriminative tariff policy adopted by the British Government, India became net exporter of raw materials and primary products.

On the other hand, it became net importer of finished goods reproduced by the British Industry.

**(ii)** Composition of exports and imports showed the backwardness of Indian economy. Exports and imports were largely restricted to Britain only due to monopoly control of India's foreign trade.

**(iii)** Surplus profit made and account of foreign trade during the British rule was distributed on administrative and as well as on war expenses. It was only used to increase the pursuits of the British Government. **Mention the demographic profile during the British rule.**

- 1) High birth and High death rate implied low survival rate, which was nearly 8 per thousand per annum.
- 2) Life expectancy was as low as 32 years which shows the lack of health care facilities, lack of awareness as well as lack of means for health care.
- 3) Literacy rate was as low as 16 percent, which reflects the social and economic backwardness of the country.
- 4) High infant Mortality rate means death rate of children below the age of one Year. It was about 218 per thousand live births.

**Mention the condition of occupation structure at the time Independence.**

- (i) Agriculture was the principal source of occupation and about 72.7 percent of working population was engaged in agriculture.
- (ii) Only 10.1% of the working population was engaged in the manufacturing sector, which showed the backwardness of Indian Industry at the time of Independence.
- (iii) Only 17.2 percent of the working population were engaged in the service sector, which also proved the slow growth of tertiary sector at the time of Independence.
- (iv) There was an unbalanced growth of Indian economy at the time of Independence

**Mention the condition of Infrastructure at the time of Independence.**

- 1) There was some infrastructural development during the British in the area of transport and communication.
- 2) Introduction of railways was a major breakthrough followed by the development of some ports and the construction of some roads.
- 3) But the main motive of the British government was to foster the interest of the British Government rather than to accelerate the growth of Indian economy.
- 4) There was transition from barter system of exchange to monetary system of exchange, which facilitated division of labour & large scale production.

**Mention the drain of Indian wealth during the colonial period.**

- 1) Drain of wealth implied all those payments which were made by India to England for which nothing was received in return .

It happened in this way:

- (1) Payments for the expenses incurred by the office set up by the colonial government in the Britian.
- (2) Payments of expenses on war fought by the British government.
- (3) Payments for the import of invisible items.

**Indicate the volume composition and direction of trade at the time of independence.**

India has been an important trading nation since ancient times. But the British government adversely affected the structure , composition and volume of India's foreign trade.

State of India's foreign trade on the eve of independence was as follows:

- 1) **Volume of Trade.** India had a large export surplus. Its export exceeds the imports and able to generate surplus of Rs 14 crore.
- 2) **Composition.** India became an exporter of primary products such as raw silk, cotton , wool, sugar indigo, jute etc. and an importer of finished consumer goods like cotton, silk and woolen clothes and capital goods like light machinery produced in the factory of Britain.
- 3) **Direction of Trade.** More than half of India's foreign trade was restricted to Britain. The rest was allowed with a few other countries like China, Sri Lanka and Persia

**Class XII**  
**Subject – Informatics Practices**

Date	Homework / Worksheet / Assignment
17-3-2020	Write answers of following questions: 1) Define term 'Computer Networking'. 2) Explain advantages of Computer Networking.
18-3-2020	Write answers of following questions: 1) Explain disadvantages of Computer Networking. 2) Explain different types of Networks.
19-3-2020	Write answers of following questions: 1) Explain following network devices: i) Modem ii) Hub iii) Switch 2) Why Switch is called Intelligent hub?
20-3-2020	Write answers of following questions: 1) Explain following network devices: i) Repeater ii) Gateway iii) Router 2) Explain the difference between Hub, Switch and Router.
21-3-2020	Write answers of following questions: 1) Define the term Topology. 2) Explain Star topology, Tree topology and Mesh topology.
23-3-2020	Write answers of following questions: 1) Define the term Internet. 2) What are the components of Internet?
24-3-2020	Write answers of following questions: 1) Explain the following with examples: i) IP Address ii) MAC Address iii) Domain name iv) Domain Name Resolution v) URL 2) Write the difference between IP address and MAC address.
25-3-2020	Write answers of following questions: 1) Explain the advantages and disadvantages of email. 2) Explain the advantages and disadvantages of chat. 3) Explain the term 'Cookies'.
26-3-2020	Write answers of following questions: 1) Define terms: i) Website ii) Webpage 2) Write the difference between a website and a webpage. 3) Write the difference between static and dynamic webpage.
27-3-2020	Write answers of following questions: 1) Define the term 'web browser'. 2) Write names of commonly used web browsers.

28-3-2020

Do the following assignment:

1. Use the following words to fill in the gaps of the text.

Share Application Scanners Data Network Communicate Printers

A computer \_\_\_\_\_ is a collection of computers that have been linked together so that they \_\_\_\_\_ with one another.

Being linked in a network enables them to \_\_\_\_\_ hardware (such as \_\_\_\_\_ and \_\_\_\_\_), software (such as \_\_\_\_\_ programs) and \_\_\_\_\_ ( such as files stored in the project folders)

2. Explain in your own words the meaning of the term “Local Area Network”

3. What two things do you need to log into a LAN?

4. What is a Wide Area Network?

5. Do you think you would share hardware or software resources in a LAN?

Why?

6. The most commonly used method to connect to a WAN is through the telephone system. To do so, what piece of equipment would you require?

7. Which of the following are true or false?

<b>T/F</b>	<b>Statement</b>
	Computers that are joined together are called a “network”
	Computers that are joined together are able to share hardware and software, but not data
	Examples of hardware that computes on a network can share are the operating system and the web browsers
	Computers that are close to one another are connected to form a LAN
	Computers in a LAN are most likely to be connected using cables
	A user name and password is normally needed to log onto a LAN
	Radio waves can be used to connect computers to a network
	The telephone system is the only way to connect computers in a WAN together
	Computers in a WAN can also be connected by satellites and fibre optic cables

30-3-2020	<p>Do the following assignment: (Multiple Choice Questions)</p> <p>1) Two devices are in network if</p> <ul style="list-style-type: none"><li>(a) a process in one device is able to exchange information with a process in another device.</li><li>(b) a process is running on both devices.</li><li>(c) the processes running off different devices are of same type.</li><li>(d) none of the mentioned.</li></ul> <p>2) What is a standalone computer?</p> <ul style="list-style-type: none"><li>(a) a computer that is not connected to a network.</li><li>(b) a computer that is being used as a server.</li><li>(c) a computer that does not have any peripherals attached to it.</li><li>(d) a computer that is used by only one person.</li></ul> <p>3) Central computer which is powerful than other computers in a network is called as _____</p> <ul style="list-style-type: none"><li>(a) Client</li><li>(b) Server</li><li>(c) Hub</li><li>(d) Switch</li></ul> <p>4) A device that forwards data packet from one network to another is called a</p> <ul style="list-style-type: none"><li>(a) Bridge</li><li>(b) Router</li><li>(c) Hub</li><li>(d) Gateway</li></ul> <p>5) Hub is a</p> <ul style="list-style-type: none"><li>(a) Broadcast device.</li><li>(b) Unicast device.</li><li>(c) Multicast device.</li><li>(d) None of the above.</li></ul> <p>6) Switch is a</p> <ul style="list-style-type: none"><li>(a) Broadcast device.</li><li>(b) Unicast device.</li><li>(c) Multicast device.</li><li>(d) None of the above.</li></ul> <p>7) The device that can operate in place of a Hub is a</p> <ul style="list-style-type: none"><li>(a) Switch</li><li>(b) Bridge</li><li>(c) Router</li><li>(d) Gateway</li></ul> <p>8) A Repeater takes a weak and corrupted signal and _____ it.</p> <ul style="list-style-type: none"><li>(a) Amplifies</li><li>(b) Regenerates</li></ul>

	(c) Re-assembles (d) Re-routes
31-3-2020	<p>Do the following assignment:</p> <p>1) Fill in the blanks:</p> <p>(a) A computer network that covers a large geographical area is called _____.</p> <p>(b) Wired networks use an access method called _____.</p> <p>(c) Wireless networks use an access method called _____.</p> <p>(d) A network of networks is known as _____.</p> <p>(e) In a network, a machine is identified by a unique address called _____.</p> <p>(f) The physical address assigned by NIC manufacturer is called _____ address.</p> <p>(g) A MAC address consumes _____ bytes or _____ bits.</p> <p>2) State True or False:</p> <p>(a) A LAN is connected to large geographical area.</p> <p>(b) A client is a computer that asks for the action in a network.</p> <p>(c) A computer is identified by 64 bit IP address.</p> <p>(d) Every object on the Internet has a unique URL.</p> <p>(e) A standalone computer may also be referred to as host.</p> <p>(f) Big networks can be of peer to peer types.</p> <p>(g) A Switch can work in place of a Hub.</p> <p>(h) A Gateway is like a Modem.</p> <p>(i) The cloud is the generic term used for Internet.</p>



## ASSIGNMENTS

Date - 16.3.2020

### Lesson - Accounting for Not for Profit Organisation

- XYZ club has a bar that maintains a separate trading account for its trading activities. Which of the following is the treatment of profit or loss on bar trading activities?
  - Profit and loss is credit in income statement
  - Profit and loss to be presented in Receipt and payment account
  - Profit and loss is added to capital fund.
  - Profit and loss to be transferred as income and expenditure A/c.
- Calculate the sports material to be debited to Income & Expenditure a/c. For the yr. ended 31-3-2007 on the basis of the following information. Amount paid for sports material during the yr. was Rs.19, 000

Particulars	1=4=2006 (Rs.)	31.3.2007 (Rs.)
Stock of sports material	7,500	6,400
Creditors for sports material	2,00	2,600

- Rs.20300
  - Rs.20700
  - Rs.20000
  - Rs.20500
- A non-profit organization received Rs.10,000 as the entrance fee of a new member. If 20% of the fee has to be capitalized, what is the amount of fee needs to be shown in the income and expenditure account?
  - Rs.9000
  - Rs.8000
  - Rs.2000
  - Rs.5000
- Prize fund Rs.10000, Interest on prize fund investments Rs.1000, Prize paid Rs.2000, Prize fund investment Rs.8000.What will be its treatment
  - Rs.20000 on liability side, Rs. 8000 on Assets side
  - Rs.1000 on liability side, Rs. 8000 on Assets side

3. Rs.1700 on liability side, Rs. 8000 on Assets side
4. Rs.9000 on liability side, Rs. 8000 on Asset.

**Date - 17.3.2020**

### **Lesson - Accounting for Not for Profit Organisation**

1. Belle, a nongovernmental not-for-profit organization, received funds during its annual campaign that were specifically pledged by the donor to another nongovernmental not-for-profit health organization. How should Belle record these funds?
  1. Increase in assets and increase in revenue
  2. Increase in assets and increase in liabilities
  3. Increase in assets and increase in deferred revenue.
  4. Decrease in assets and decrease in fund balance.
2. Not-for-profit organisations have some distinguishing features from that of profit organisations. State any one of them.
3. What is the capital of a Non-Profit Organization generally known as?
4. Name any two accounts required to be prepared in Financial Statements by Not-For-Profit Organizations at the end of the year.
5. State the main aim of a not-for-profit organisation.
6. Write any four features of Receipt and Payment Account?

**Date - 18.3.2020**

### **Lesson - Accounting for Not for Profit Organisation**

1. Calculate the amount of stationery to be posted to Income and Expenditure Account of Indian Cultural Society for the year ending 31<sup>st</sup> March, 2018 from the following information :

<b>Particulars</b>	<b>1.4.2017 (Rs.)</b>	<b>31.3.2018 (Rs.)</b>
Stock of stationery	21,000	18,000
Creditors for stationery	11,000	23,000

2. Stationery purchased during the year ended 31<sup>st</sup> March 2018 was Rs.75,000. Also, present the relevant items in the Balance Sheet of the society as at 31<sup>st</sup> March 2018.
3. From the following information, calculate the amount of subscriptions to be credited to the income and expenditure account for the year 2007—08.

<b>Particulars</b>	<b>Amt (Rs.)</b>
Subscriptions received during the year	50,000
Subscriptions outstanding on 31st March, 2007	20,000
Subscriptions outstanding on 31st March, 2008	6,000
Subscriptions received in advance on 31st March, 2007	8,000
Subscriptions received in advance on 31st March, 2008 Subscriptions of Rs. 1,500 are still in arrears for the year 2006-07.	9,000

**Date - 19.3.2020**

## **Lesson - Accounting for Not for Profit Organisation**

### **QUESTION 1**

Mention the financial statements of the Not-for-Profit Organization.

### **QUESTION 2**

Non-for-Profit Organisation prepare

- (a) Income and Expenditure account
- (b) Trading and Profit & Loss account
- (c) Only the trading account
- (d) None of the above

### **QUESTION 3**

Receipt and payments account is a summary of

- (a) Debit & Credit balance of Ledger account
- (b) Cash receipts & payment
- (c) Income and Expenses
- (d) A balance of assets and liabilities

### **QUESTION 4**

Subscription received in advance by a club are shown

- (a) In the credit side of the income and expenditure account
- (b) In the asset side of the balance sheet
- (c) In the liabilities side of the balance sheet
- (d) None of the above

### **QUESTION 5**

Donation received for a special purpose is a

- (a) Liability

- (b) Revenue Receipt
- (c) Capital Receipt
- (d) None of the above

**QUESTION 6**

Receipt and Payment account is

- (a) Nominal Account
- (b) Real Account
- (c) Personal Account
- (d) None of the above

**QUESTION 7**

Subscription received in advance during the current year is

- (a) An income
- (b) An asset
- (c) A liability
- (d) None of the above

**20.3.2020**

**Lesson - Accounting for Not for Profit Organisation**

**QUESTION 1**

If there is a match fund, then match expenses and match income are transferred to

- (a) Income and Expenditure Account
- (b) An assets side of Balance Sheet
- (c) Liabilities of the Balance Sheet
- (d) None of the above

**QUESTION 2**

Show the format of receipt and payments account.

**Date - 21.3.2020**

1. Give two examples of not for profit organisation
2. State the main aim of Not for profit organisation
3. Give two main sources of income for Not for profit organisation
4. State two characteristics of Not for profit organisation.
5. What are membership subscriptions?

23.3.2020

### Lesson - Accounting for Not for Profit Organisation

1. From the following Receipts and Payments Accounts of Cricket Club and the additional information given, prepare the Income and Expenditure Account for the Year ending 31-12-2018 and Balance sheet as on that date:

#### RECEIPTS AND PAYMENTS ACCOUNT for the year ending 31-12-2018

To bal. b/d	Rs.		Rs.
-Cash	3520	By Maintenance	6820
-Bank	27380	By Crockery Purchased	2650
-Fixed Deposit @ 6%	30000	By Match Expenses	13240
To Subscription (including Rs. 6000 for 2017)	40000	By Salaries	11000
TO Entrance fees	2750	By Conveyance	820
To Donation	5010	By Upkeep of Lawns	4240
To Interest on Fixed Deposits	900	By postage stamps	1050
To Tournament Fund	20000	By Purchase Of cricket goods	9720
To Sale of Crockery(book value Rs. 1200)	2000	By Sundry expenses	2000
		By Investments	5700
		By Tournament Expenses	18800
		By balance c/d:	
		-Cash	2200
		-Bank	23320

		Fixed Deposits	30000
	<b>131560</b>		<b>131560</b>

Additional Information:

1. Salary outstanding is Rs. 1000.
2. Opening Balance of Stock of Postage and Stationery and Cricket goods is Rs. 750 and Rs. 3210 respectively. Closing stock of the same is Rs. 900 and Rs. 2800 respectively.
3. Outstanding subscription for 2017 and 2018 is Rs. 6600 and Rs. 8000 respectively.

2. Receipt and Payment Account of Shankar Sports club is given below, for the

<b>Receipt and Payment Account for the year ending March 31, 2017</b>			
<b>Receipts</b>	<b>Amount Rs</b>	<b>Payments</b>	<b>Amount Rs</b>
Opening Cash in hand	2,600	Rent	18,000
Entrance fees	3,200	Wages	7,000
Donation for building	23,000	Billiard table	14,000
Locker rent	1,200	Furniture	10,000
Life membership fee	7,000	Interest	2,000
Profit from entertainment	3,000	Postage	1,000
Subscription	40,000	Salary	24,000
	.....	Cash in hand	4,000
	80,000		80,000

year ended March 31, 2017

Prepare Income and Expenditure Account and Balance Sheet with help of following Information:

Subscription outstanding on March 31, 2016 is Rs 1, 200 and Rs 2,300 on March 31, 2017, opening stock of postage stamps is Rs 300 and closing stock is Rs 200, Rent Rs 1,500 related to 2015 and Rs 1,500 is still unpaid. On April 01, 2016 the club owned



furniture Rs 15,000, Furniture valued at Rs 22,500. On March 31, 2016. The club took a loan of Rs 20,000 (@ 10% p.a.)

**Date - 24.3.2020**

1. What is meant by Endowment Fund?
2. How are Not for profit organisations organized?
3. Name two Financial statements required to be prepared by not for profit organisations at the end of the year.
4. Give two point of difference between a Cash book and Receipt and Payments Account.
5. Give on similarity between Receipt and Payments Account and Income and Expenditure Account.

Date - 25.3.2020

**Ques - From the following Receipts and Payments Account and additional information, prepare Income and Expenditure Account and Balance Sheet of Sears Club, Noida as on March 31, 2018.**

**Receipts and Payments & Account of Sears Club for the year ended 31-3-2018**

Receipts	Amount (₹)	Payments	Amount (₹)
To Balance b/d	20,000	By Stationery	23,400
To Subscriptions		By 12% Investments	8,000
2016-17   40,000		By Electricity expenses	10,600
2017-18   94,000		By Expenses on lectures	30,000
2018-19 <u>7,200</u>	1,41,200	By Sports equipment	59,000
To Donations for building	40,000	By Books	40,000
To Interest on Investments	800	By Balance c/d	50,000
To Government Grant	17,400		
To Sale of old furniture (Book value ₹ 4,000)	1,600		
	<b>2,21,000</b>		<b>2,21,000</b>

**Additional Information :**

**(i) The club has 200 members each paying an annual subscription of Rs. 1,000. Rs. 60,000 were in arrears for last year and 25 members paid in advance in the last year for the current year.**

**(ii) Stock of stationery on 1-4-2017 was Rs. 3,000 and on 31-3-2018 was Rs. 4,000.**

**Date 26.3.2020**

**Ques -- Namanjyot Society showed the following position :  
Balance Sheet as at 31st March, 2018**

<b>Liabilities</b>	<b>Amount</b>	<b>Assets</b>	<b>Amount</b>
Subscriptions received in advance	6,000	Cash at Bank	30,000
Capital Fund	72,000	Cash in hand	8,000
		Furniture	40,000
	78,000		78,000

**Receipts and Payments Account for the year ending 31st March, 2018**

Receipts	Amount ₹	Payments	Amount ₹
To Balance b/d		By Computers (1.10.2018)	1,00,000
Cash at Bank    30,000		By Office Expenses	29,000
Cash in Hand <u>24,000</u>	54,000	By Electric Charges	15,000
To Sale proceeds of old newspapers	900	By Postage and Stationery	9,000
To Locker's Rent	7,000	By 10% Investments (on 1.12.2017)	60,000
To Interest on Investments	1,600	By Balance c/d	
To Entrance Fees	50,000	Cash at Bank   80,000	
To Life Membership Fees	1,00,000	Cash in Hand <u>35,500</u>	1,15,500
To Membership subscriptions	98,000		
To Subscriptions for relief fund	17,000		
	3,28,500		3,28,500

***Additional Information :***

**(i) Computers were to be depreciated @ 60% p.a. and furniture @ 10% p.a.**

**(ii) Membership subscription included < 20,000 received in advance.**

**(iii) Electric charges outstanding < 10,000.**

**Prepare Income and Expenditure Account for the year ending 31st March, 2018.**

**Date - 27.3.2020**

**Ques - What is meant by 'Life membership fees' ?**

**Ques - From the following Receipts and Payments Account and additional information, prepare Income and Expenditure Account and Balance Sheet of Sears Club, Noida as on March 31, 2018.**

**Receipts and Payments & Account of Sears Club for the year ended 31-3-2018**

Receipts	Amount (₹)	Payments	Amount (₹)
To Balance b/d	20,000	By Stationery	23,400
To Subscriptions		By 12% Investments	8,000
2016-17   40,000		By Electricity expenses	10,600
2017-18   94,000		By Expenses on lectures	30,000
2018-19 <u>7,200</u>	1,41,200	By Sports equipment	59,000
To Donations for building	40,000	By Books	40,000
To Interest on Investments	800	By Balance c/d	50,000
To Government Grant	17,400		
To Sale of old furniture (Book value ₹ 4,000)	1,600		
	<b>2,21,000</b>		<b>2,21,000</b>

**Additional Information :**

**(i) The club has 200 members each paying an annual subscription of Rs. 1,000. Rs. 60,000 were in arrears for last year and 25 members paid in advance in the last year for the current year.**

**(ii) Stock of stationery on 1-4-2017 was ₹ 3,000 and on 31-3-2018 was Rs. 4,000.**

## Question 2

Domex Club disclosed that it received ₹ 1,30,000 by way of subscription during 2016.

<i>Additional Information:</i>	(₹)
Subscription outstanding 2015 (of which ₹ 3,000 received during 2016)	4,000
Advance subscription 2015	3,500
Advance subscription 2016	2,400
Subscription due for the year 2016	1,500

Show how the subscriptions will appear in Income and Expenditure Account of 2016 and in Balance Sheet of that year.

## Question 3

(i) Show the following information in the final accounts of Lion's Club as on 31st March, 2017:

Club as on 31st March, 2017:	(₹)
Sports Fund	50,000
Sports Expenses	64,000
Donation for Sports Fund	10,000
Sale of Tickets of Sports Match	8,000

(ii) What will be the impact if sports expenses increase by ₹ 9,000 in case (i)?

28.3.2020

Question 1

From the following extracts of the Receipts and Payments Account and the additional information, you are required to compute the income from subscription for the year ended March 31, 2017 and show the subscription item in the final account of the club.

**Receipts and Payments Account**  
*for the year ended 31st March, 2017*

Dr.	(₹)	Cr.	(₹)
Receipts		Payments	
Subscriptions	10,000		

**Additional Informations:**

(i) Subscription outstanding on 31st March, 2016	2,000
(ii) Subscription outstanding on 31st March, 2017	4,000
(iii) Subscription received in advance on 31st March, 2016	3,000
(iv) Subscription received in advance on 31st March, 2017	2,000

Question 2

On the basis of the information given below, calculate the amount of stationery to be debited to the Income and Expenditure Account of Good Health Sports Club for the year ended 31st March, 2017.

Particulars	1-4-2016 (₹)	31-3-2017 (₹)
Stock of Stationery	8,000	6,000
Creditors for Stationery	9,000	11,000

Stationery purchased during the year ended 31-3-2017 was ₹ 47,000.

30.3.2020

**Question 1**

Prepare Income and Expenditure A/c from the following particulars of Youth Club for the year ended on 31st March, 2018:

**Receipts and Payments Accounts**  
for the year ended 31st March, 2018

Receipts	(₹)	Payments	(₹)
To Balance b/d	32,500	By Salaries	31,500
To Subscription		By Postage	1,250
2016-17                   1,500		By Rent	9,000
2017-18                   60,000		By Printing and Stationery	14,000
2018-19 <u>1,800</u>	63,300	By Sports Materials	11,500
To Donations (Billiards Table)	90,000	By Miscellaneous Expenses	3,100
To Entrance Fees	1,100	By Furniture (1-10-2017)	20,000
To Sale of Old Magazines	450	By 10% Investment (1-10-2017)	70,000
		By Balance c/d (31-3-2018)	27,000
	<u>1,87,350</u>		<u>1,87,350</u>

**Additional Informations:**

- (i) Subscription outstanding as at March 31st 2018, ₹ 16,200.
- (ii) ₹ 1,200 is still in arrears for the year 2016-17 for subscription.
- (iii) Value of sports material at the beginning and at the end of the year was ₹ 3,000 and ₹ 4,500 respectively.
- (iv) Depreciation to be provided @ 10% p.a. on furniture.

31.3.2020

**Question 1**

A club has 1,000 members. Each member has to pay subscription @ ₹ 50. Total subscription received during the year ending 2016 was ₹ 90,000 including ₹ 23,000 for the year 2015 and ₹ 24,000 for the year 2017. Twenty five members paid their subscription for 2016 in the year 2015.

Compute the amount of subscription to be credited to Income and Expenditure Account and outstanding subscription to be shown in Balance Sheet.



## Question 2

Calculate the amount of sports material to be transferred to Income and Expenditure account of Raman Bhalla Sports Club, Ludhiana, for the year ended 31st March, 2018:

<i>Particulars</i>	<i>Amount (₹)</i>
(i) Sports Material sold during the year (Book Value ₹ 50,000)	56,000
(ii) Amount paid to creditors for sports material	91,000
(iii) Cash purchase of sports material	40,000
(iv) Sports material as on 31-3-2017	50,000
(v) Sports material as on 31-3-2018	55,000
(vi) Creditors for sports material as on 31-3-2017	37,000
(vii) Creditors for Sports material as on 31-3-2018	45,000



**Class XII**  
**Subject – Informatics Practices**

Date	Homework / Worksheet / Assignment
17-3-2020	Write answers of following questions: 1) Define term 'Computer Networking'. 2) Explain advantages of Computer Networking.
18-3-2020	Write answers of following questions: 1) Explain disadvantages of Computer Networking. 2) Explain different types of Networks.
19-3-2020	Write answers of following questions: 1) Explain following network devices: i) Modem ii) Hub iii) Switch 2) Why Switch is called Intelligent hub?
20-3-2020	Write answers of following questions: 1) Explain following network devices: i) Repeater ii) Gateway iii) Router 2) Explain the difference between Hub, Switch and Router.
21-3-2020	Write answers of following questions: 1) Define the term Topology. 2) Explain Star topology, Tree topology and Mesh topology.
23-3-2020	Write answers of following questions: 1) Define the term Internet. 2) What are the components of Internet?
24-3-2020	Write answers of following questions: 1) Explain the following with examples: i) IP Address ii) MAC Address iii) Domain name iv) Domain Name Resolution v) URL 2) Write the difference between IP address and MAC address.
25-3-2020	Write answers of following questions: 1) Explain the advantages and disadvantages of email. 2) Explain the advantages and disadvantages of chat. 3) Explain the term 'Cookies'.
26-3-2020	Write answers of following questions: 1) Define terms: i) Website ii) Webpage 2) Write the difference between a website and a webpage. 3) Write the difference between static and dynamic webpage.
27-3-2020	Write answers of following questions: 1) Define the term 'web browser'. 2) Write names of commonly used web browsers.

28-3-2020

Do the following assignment:

1. Use the following words to fill in the gaps of the text.

Share Application Scanners Data Network Communicate Printers

A computer \_\_\_\_\_ is a collection of computers that have been linked together so that they \_\_\_\_\_ with one another.

Being linked in a network enables them to \_\_\_\_\_ hardware (such as \_\_\_\_\_ and \_\_\_\_\_), software (such as \_\_\_\_\_ programs) and \_\_\_\_\_ ( such as files stored in the project folders)

2. Explain in your own words the meaning of the term “Local Area Network”

3. What two things do you need to log into a LAN?

4. What is a Wide Area Network?

5. Do you think you would share hardware or software resources in a LAN?

Why?

6. The most commonly used method to connect to a WAN is through the telephone system. To do so, what piece of equipment would you require?

7. Which of the following are true or false?

<b>T/F</b>	<b>Statement</b>
	Computers that are joined together are called a “network”
	Computers that are joined together are able to share hardware and software, but not data
	Examples of hardware that computes on a network can share are the operating system and the web browsers
	Computers that are close to one another are connected to form a LAN
	Computers in a LAN are most likely to be connected using cables
	A user name and password is normally needed to log onto a LAN
	Radio waves can be used to connect computers to a network
	The telephone system is the only way to connect computers in a WAN together
	Computers in a WAN can also be connected by satellites and fibre optic cables

30-3-2020	<p>Do the following assignment: (Multiple Choice Questions)</p> <p>1) Two devices are in network if</p> <ul style="list-style-type: none"><li>(a) a process in one device is able to exchange information with a process in another device.</li><li>(b) a process is running on both devices.</li><li>(c) the processes running off different devices are of same type.</li><li>(d) none of the mentioned.</li></ul> <p>2) What is a standalone computer?</p> <ul style="list-style-type: none"><li>(a) a computer that is not connected to a network.</li><li>(b) a computer that is being used as a server.</li><li>(c) a computer that does not have any peripherals attached to it.</li><li>(d) a computer that is used by only one person.</li></ul> <p>3) Central computer which is powerful than other computers in a network is called as _____</p> <ul style="list-style-type: none"><li>(a) Client</li><li>(b) Server</li><li>(c) Hub</li><li>(d) Switch</li></ul> <p>4) A device that forwards data packet from one network to another is called a</p> <ul style="list-style-type: none"><li>(a) Bridge</li><li>(b) Router</li><li>(c) Hub</li><li>(d) Gateway</li></ul> <p>5) Hub is a</p> <ul style="list-style-type: none"><li>(a) Broadcast device.</li><li>(b) Unicast device.</li><li>(c) Multicast device.</li><li>(d) None of the above.</li></ul> <p>6) Switch is a</p> <ul style="list-style-type: none"><li>(a) Broadcast device.</li><li>(b) Unicast device.</li><li>(c) Multicast device.</li><li>(d) None of the above.</li></ul> <p>7) The device that can operate in place of a Hub is a</p> <ul style="list-style-type: none"><li>(a) Switch</li><li>(b) Bridge</li><li>(c) Router</li><li>(d) Gateway</li></ul> <p>8) A Repeater takes a weak and corrupted signal and _____ it.</p> <ul style="list-style-type: none"><li>(a) Amplifies</li><li>(b) Regenerates</li></ul>

	(c) Re-assembles (d) Re-routes
31-3-2020	<p>Do the following assignment:</p> <p>1) Fill in the blanks:</p> <p>(a) A computer network that covers a large geographical area is called _____.</p> <p>(b) Wired networks use an access method called _____.</p> <p>(c) Wireless networks use an access method called _____.</p> <p>(d) A network of networks is known as _____.</p> <p>(e) In a network, a machine is identified by a unique address called _____.</p> <p>(f) The physical address assigned by NIC manufacturer is called _____ address.</p> <p>(g) A MAC address consumes _____ bytes or _____ bits.</p> <p>2) State True or False:</p> <p>(a) A LAN is connected to large geographical area.</p> <p>(b) A client is a computer that asks for the action in a network.</p> <p>(c) A computer is identified by 64 bit IP address.</p> <p>(d) Every object on the Internet has a unique URL.</p> <p>(e) A standalone computer may also be referred to as host.</p> <p>(f) Big networks can be of peer to peer types.</p> <p>(g) A Switch can work in place of a Hub.</p> <p>(h) A Gateway is like a Modem.</p> <p>(i) The cloud is the generic term used for Internet.</p>

## **TOPIC: NOTICE WRITING**

**Date: 17 March, 2020**

### ➤ **Introduction**

A notice is a formal means of communication. The purpose of a notice is to announce or display an information to a specific group of people. Notices are generally mean to be pinned up on specific display boards whether in schools or in public places. Notices issued by the government appear in newspapers.

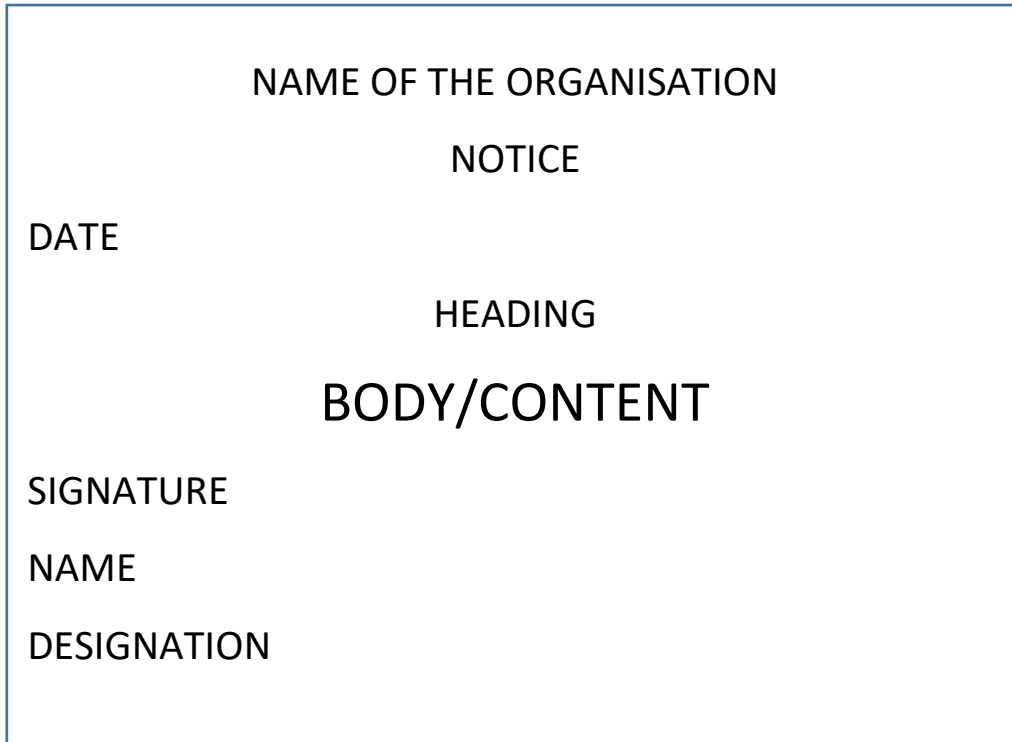
### ➤ **Some Useful Tips**

1. A well written notice must inform the readers about he 5Ws, i.e.,
  - a. What is going to happen? (event)
  - b. Where it'll take place? (venue)
  - c. When it'll take place? (date/time)
  - d. Who can apply or is eligible for it?
  - e. Whom to contact? (issuing authority)
2. Only the most important points should be written.
3. A.O.D i.e., any other details given in the question.
4. One is free to add any relevant information, not included in the question.
5. The sentences should be in passive voice as far as possible.
6. The sentences should be short and grammatically accurate.
7. The notice should be presented within a box.
8. The word limit of a notice should be only 40-50 words.

### ➤ **Format of a Notice**

A notice should be written in the given format:

1. Name of the organisation issuing the notice (in BLOCK letters)
2. The title 'NOTICE'
3. Date (on left side)
4. Heading to introduce the subject of the notice (in BLOCK letters)
5. The body of the notice
6. Writer's signature
7. Writer's name (in BLOCK letters)
8. Writer's designation



### TYPES OF NOTICES





Date: 18 March, 2020

**Sample questions:**

1. As the manager of Hotel Excelsior, write a notice for the guests staying in the hotel cautioning them about a lift that has gone out of order. You are Rahul/Radhika.

<p>HOTEL EXCELSIOR, MUSSOORIE</p> <p>NOTICE</p> <p>March 18, 2020</p> <p>ATTENTION ESTEEMED GUESTS</p> <p>We regret to inform you that the hotel lift will be shut down for 4 hrs. (10 am-2 pm) for necessary repairs to fix a fault in the cable. We deeply regret the inconvenience caused to you &amp; would like to request you to use the service lift instead.</p> <p><i>Rahul</i></p> <p>RAHUL</p> <p>Manager</p>
--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------

2. As the honorary director of RAHAT- an Old Age Home, write a notice of appeal to be published in newspapers asking donation of man hours from general public.

<p>RAHAT- AN OLD AGE HOME</p> <p>NOTICE</p> <p>March 18, 2020</p> <p>DONATE TIME FOR A NOBLE CAUSE!</p> <p>The elders in our old age home need your help. Spend whatever time you can spare with them. Donate your precious time for this unique cause. For more details call us on toll free at 1800-272-xxxx.</p> <p><i>Virat Kheka</i></p> <p>VIRAT KHEKA</p> <p>Honorary Director</p>
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Date: 19 March, 2020

**Practice Questions:**

- 1. You are Srinivas / Srinidhi of D.P. Public School, Nagpur . As a student Editor of your school magazine, draft a notice in not more than 50 words for your school notice board inviting articles / sketches from the students of all classes.**
- 2. You are Rohit / Ritu , Secretary, Welfare Association, ABC Colony, Chennai, Write a notice in not more than 50 words to be placed on the notice board informing the residents that there would be no water supply for two days in your colony due to major pipeline repair work.**

## **TOPIC: POSTER DESIGNING**

**Date: 20 March, 2020**

### ➤ **Kinds of Poster**

#### **1) Event Poster**

It includes events like school fest, book fairs, blood donation camps, etc. It deals with events that have to take place. The event posters contain following:

- Slogan
- Statement
- Venue
- Date
- Special feature
- Name of the organiser

#### **2) Non-Event Poster**

It includes posters on social issue, social events and based habits (child marriage, female foeticide, alcohol abuse, etc.). This type of poster deals with important everyday school issues also. The non-event posters contain following:

- Slogan
- Statement
- List of reasons
- Name of person

### ➤ **Drafting a Poster**

- Give a catchy title or slogan.
- Graphics in the form of matchstick figures.
- Date, time, venue must be mentioned, if the poster is being drafted for an event.
- Name of the organisation which has issued the poster.
- 'Sponsored by' to be post scripted at the bottom.



Date: 23 March, 2020

**Practice Questions:**

- 1. Water is precious and each one of us must stop wastage. Prepare a poster in not more than 50 words, for creating that awareness.**
  
- 2. Your school is planning a campaign in support of eye donation to mobilize the students and society. Design a poster to be displayed in different areas of the locality surrounding your school highlighting the need for eye donation and eye banks.**

## TOPIC: LETTER WRITING

Date: 24 March, 2020

Letter writing is an important channel of communication between people who are geographically distinct from one other, in general there are two types of letters.

### LETTERS



#### ➤ **Formal Letters**

They are the letters written to convey official business and information. These are sent out when we need to write to various public bodies and agencies for our environment in civic life.

#### ➤ **Informal Letters**

They are the personal letters to communicate with friends and family.

### FORMAL LETTERS

These include business/official letters (for making an inquiry, registration, complaints, placing orders and sending replies), letters to editor (giving suggestion on any issue) and application for job.

#### ➤ **Steps to write formal letters**

- **i)** Introducing oneself if it is a first time you are writing.
- **ii)** Referring to an earlier letter if you are responding to it.
- Stating the purpose of the letter.
- **i)** Stating action required from the addressing.
- **ii)** Explaining actions
- **i)** Arguing action to be taken

ii) Offering assistances in future.

➤ **Value Points**

- a. Person to whom letter is addressed
- b. Tone you should adopt
- c. Completeness of the message
- d. Action required

➤ **Parts of a letter**

1. Sender's address
2. Date
3. Receiver's address
4. Salutation/Subject
5. Subject/Salutation
6. Body of the letter
7. Complementary Ending

Sender's Address

Date

Receiver's Address

Subject

Salutation

Body of the Letter

(in about 3-4 paragraphs)

Yours Truly/Faithfully

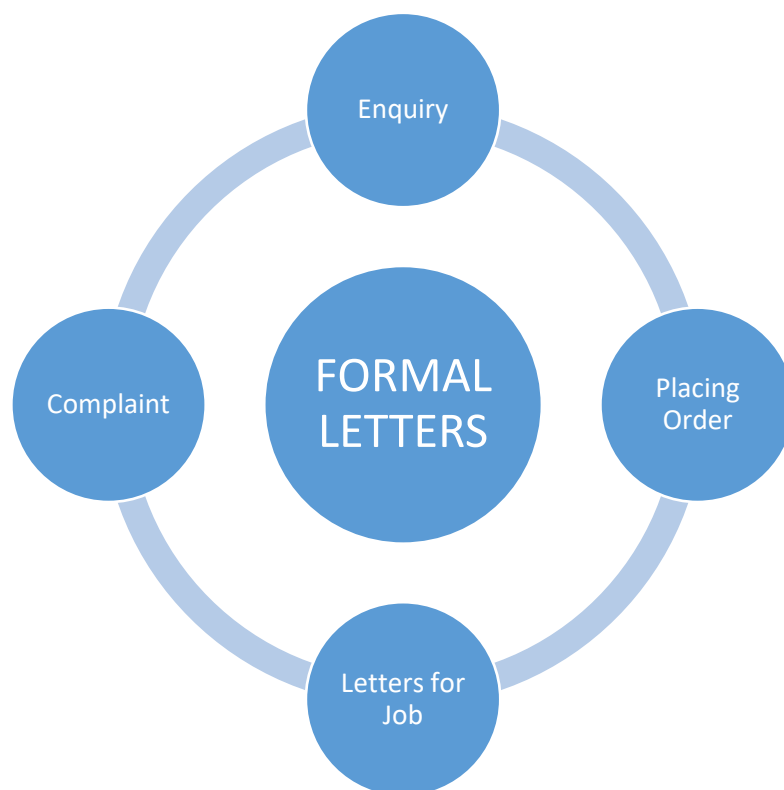
Signature

Name

designation

➤ **Points to remember**

- The date and signature are very important.
- Don't use commas in address.
- Always end your letter politely with a complementary close.





Date: 25 March, 2020

**Sample Questions:**

- 1. Write a letter to M/s. Oxford Publishing House, London complaining that the books sent by them were not those you had ordered for. Ask for replacement. You are Varun Joshi, Sector-20, Chandigarh.**

Examination Hall  
Sector-20, Chandigarh

March 25, 2020

M/s. Oxford Publishing House  
Consumer Complaint Division  
London

Subject- Complaint regarding receipt of wrong set of books.

Dear Sir

On February 11, 2020 I bought a book set (Order No. 000154) to be delivered to Chandigarh, Sector-20.

To my dismay, I have not received the set I ordered for and have instead, received the wrong book set. I am highly disappointed.

To resolve the problem, I would appreciate it if you could replace the wrong book set with the one originally ordered.

Enclosed are copies of the transaction document and the receipt.

I look forward to your reply and a resolution to my problem and will wait until the aforementioned time before seeking help from a consumer protection agency or the Better Business Bureau. Please contact me at the above address or by phone at 098100XXXXX.

Yours Truly

*Varun Joshi*

Varun Joshi

2. You are Mohan/Monica, Librarian of DPS, Mumbai. You have been asked to place an order for some books. Write the letter to the Manager, Bhavan Booksellers, Mumbai.

DPS  
Mumbai

March 25, 2020

The Manager  
Bhavan Booksellers  
Mumbai  
Subject – Placing order for books

Dear Sir

I've reliably learnt that your book shop supplies books at discount rates. I would like to place the following order:

S.No.	Book Name	Author Name	Quantity
1.	English	NCERT	5
2.	Informatics Practices	NCERT	5
3.	Accountancy	D.K. Goel	6
4.	Economics	Sandeep Garg	4
5.	B.Studies	Subhash Dey	6

The payment will be made by cheque on delivery of the books. Kindly ensure that the order reaches us within a week

Yours truly

*Mohan*

Mohan

Date: 26 March, 2020

**Practice Questions:**

- 1. You are Megha/Maya of class XII of St. Peter School, Pitampura, Delhi. Write a letter to the Manager of Book World, RK Puram, New Delhi inquiring about the availability of few books you need for your school library. Write as secretary of your school library.**
- 2. As a regular commuter by bus from Noida to Delhi, you have been witnessing rash driving by the bus drivers daily without an exception. Write a letter to the Editor, 'The Times of India' drawing the attention of the General Manager, Delhi Transport Corporation to this problem. Your are Priti/Prakash, 15 Udyog Vihar, Noida.**

Date: 27 March, 2020

**Practice Questions:**

- 1. You are Amit, 513, MG Marg, Delhi. You have seen an advertisement in Hindustan Times for the post of Marketing Manager. Write a letter to General Manager, JP Pvt. Ltd., Mumbai with complete bio data.**
- 2. As Mr. R. Singh, Head of the Department of Chemistry, Cambridge High School, Pune, you had placed an order with Messrs. Scientific Equipments, Dadar, Mumbai for test tubes and jar for the lab. When the parcel was received you observed that markings on the test tubes were not clear and some of the jars were damaged. Write a letter of complaint seeking immediate replacement.**

Date: 28 March, 2020

## **TOPIC: ADVERTISEMENTS**

Advertisements refers to a short composition in the form of a notice, picture or film telling people about a product, job or a service. It is generally inserted in the newspaper or displayed on the T.V. It is a very powerful means of promoting sales of goods.

### ➤ **Newspaper Advertisements**

These are of 2 types:

1. Classified Advertisements
2. Display Advertisements

### ➤ **Classified Advertisements**

- These are brief advertisements which are inserted in the classified section of newspaper.
- For ex: situation vacant, To-let, etc.
- Grammatical accuracy is hardly needed.
- This is done in order to convey maximum information in minimum words.
- They are written in brief.

### ➤ **Display Advertisements**

This is a form of advertising that conveys a commercial message visually using text, logos, animations, photos, videos or other graphic techniques.

### ➤ **Types of Classified Advertisements**

- Situation Vacant/Wanted
- To-Let
- Accommodation Wanted
- Sale/Purchase (any property or household goods)
- Matrimonial
- Education
- Lost and Found
- Missing
- Tutor Wanted

Date: 30 March, 2020

**Sample Questions:**

1. **Your pet dog Russian Alsatian is missing. Draft an advertisement for the same.**

**MISSING PET**

Pet dog missing Russian Alsatian, 2.5 yrs. Old, well built, brown in colour, white spot on nose, since March 27, 2020, near Central Green Park, Faridabad.

Contact: Mr. ABC

Ph. No.: XXXXXX6763

2. **You found a bag full of new clothes. Draft an advertisement for the same.**

**FOUND**

Found a bag of newly stitched clothes, unstitched dress materials, a fashion design book and some cash. Whosoever has lost it may claim by identifying them and by providing authentic proof of ownership. If nobody claims the bag it'll be handed over to Government Vocational Training Institute for Orphans, James Kirkup Road, Kovalum for proper use.

Contact: Mr. XYZ

Ph. No.: XXXXXX9539

Date: 31 March, 2020

**Practice Questions:**

- 1. You propose to sell your flat as you are going abroad. Draft an advertisement to be published in the classified columns of 'The Times of India', New Delhi. Invent necessary details.**
- 2. You are Manisha. You have started hobby classes for children of 6 to 12 years. Prepare a suitable advertisement giving all the required details.**

**16th - 17th MARCH 2020**

**CLASS – XII MATHEMATICS ASSIGNMENT NO. 1 MATRICES**

Q1(i) If a matrix has 12 elements, what are the possible orders it can have? What if it has 7 elements?

(ii) If a matrix has 8 elements, what are the possible orders it can have? What if it has 5 elements?

Q2. Construct a 2x3 matrix whose elements in the  $e^{\text{th}}$  row and  $j^{\text{th}}$  column is given by :-

(i)  $a_{ij} = \frac{i+3j}{2}$  (i)  $a_{ij} = \frac{2i+3j}{2}$  (iii)  $a_{ij} = \frac{3i+j}{2}$  (iv)  $a_{ij} = \frac{3i-j}{2}$

Q3. Construct a 4x3 matrix whose elements are:-

(i)  $a_{ej} = 2i + \frac{e^j}{f}$  (ii)  $a_{ej} = \frac{i-j}{j+j}$  (iii)  $a_{ej} = i$

Q4. If  $\begin{pmatrix} 2+3 & 2+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z=2c \end{pmatrix} = \begin{pmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c-2 \\ 2b+4 & -21 & 0 \end{pmatrix}$

Obtain the values of a, b, c, x, y and z.

Q5. Find matrices x and y i.e.

$2x-y = \begin{pmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{pmatrix}$  and  $x+2y = \begin{pmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{pmatrix}$

Q6. Find the value of x such that :-

$\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$

Q7. If  $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$  prove that  $A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$  where n is any positive integer.

Q8. If  $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{pmatrix}$ , find  $A^2 - 4A + 3I_3$

Q9. Express the matrix  $A = \begin{pmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{pmatrix}$  as the sum of a symmetric and a skew symmetric matrix

Q10. Express the following matrices as the sum of symmetric and skew-symmetric matrices:-

(i)  $A = \begin{pmatrix} 6 & 1 \\ 3 & 4 \end{pmatrix}$  (ii)  $A = \begin{pmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{pmatrix}$

(iii)  $A = \begin{pmatrix} 2 & 3 & -1 \\ -1 & 4 & 2 \\ 6 & 0 & 8 \end{pmatrix}$  (iv)  $\begin{pmatrix} 6 & 1 & -5 \\ -2 & -5 & 4 \\ -3 & 3 & -1 \end{pmatrix}$

Q11. Using elementary transformation, find the inverse of the following matrices :-

(i)  $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$  (ii)  $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$  (iii)  $\begin{pmatrix} 2 & -3 & 3 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{pmatrix}$

(iv)  $\begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{pmatrix}$

Determinants

Evaluate

1.  $\begin{pmatrix} 9 & 9 & 12 \end{pmatrix}$  2.  $\begin{pmatrix} 265 & 240 & 219 \end{pmatrix}$  3.  $\begin{pmatrix} 3 & -4 & 5 \end{pmatrix}$



$$4. \begin{pmatrix} 1 & -3 & -4 \\ 1 & 9 & 12 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{pmatrix} \quad \begin{matrix} 240 & 225 & 198 \\ 219 & 198 & 181 \end{matrix} \quad \begin{matrix} 1 & 1 & -2 \\ 2 & 3 & 1 \end{matrix}$$

Using properties of determinants, prove that :-

$$1. \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a) \quad 2. \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

$$3. \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1-x & -1 \\ 1 & 1 & 1+y \end{vmatrix} = x y \quad 4. \begin{vmatrix} a+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 2 & 1 \end{vmatrix} = -2(x^3+y^3)$$

$$5. \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} = -2(x^3+y^3) \quad 6. \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$$

$$7. \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$$

$$8. \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x), \text{ where } p \text{ is a scalar.}$$

Q9. Find the area of the triangle whose vertices are A(-2, -3), B(3,2) and C (-1, -8)

Q10. Show that the pts A(a+b+c), B(b+c+a) and C (c+a+b) are collinear. Find the inverse of each of the matrices given below :-

$$Q11. \begin{vmatrix} 3 & -5 \\ -1 & 2 \end{vmatrix} \quad Q12. \begin{vmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{vmatrix} \quad Q13. \begin{vmatrix} 2 & -1 & -1 \\ 3 & 0 & -1 \\ 2 & 6 & 0 \end{vmatrix}$$

$$Q14. \begin{vmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{vmatrix} \quad Q15. \begin{vmatrix} 8 & -4 & 1 \\ 10 & 0 & -6 \\ 8 & 1 & 6 \end{vmatrix}$$

$$Q16. \begin{matrix} 5x+2y=4 \\ 7x+3y=5 \end{matrix} \quad Q17. \begin{matrix} 3x+4y-5=0 \\ x-y+3=0 \end{matrix} \quad Q18. \begin{matrix} 3x-2y+3z=8 \\ 2x+y-z=1 \\ 4x-3y+2z=4 \end{matrix}$$

$$Q19. \begin{matrix} x-y+z=-1 \\ 2x+y-z=2 \\ x-2y-z=4 \end{matrix} \quad Q20. \begin{matrix} x+y+z=4 \\ 2x-y+z=-1 \\ 2x+y-3=-9 \end{matrix} \quad Q21. \begin{matrix} 4x+2y+3z=5 \\ x-2y+z=-4 \\ 3x-y-2z=3 \end{matrix}$$

$$Q22. \begin{matrix} 4x+2y+3z=9 \\ X+y+z=1 \\ 3x+y-2z=1 \end{matrix} \quad Q23. \begin{matrix} 3x-4y+2z=-1 \\ 2x+3y+5z=7 \\ x+z=2 \end{matrix} \quad Q24. \begin{matrix} 6x-9y-20z=-4 \\ 4x-15y+10z=-1 \\ 2x-3y-5z=-1 \end{matrix}$$

Q25.  $5x - y = 7$

$$2x + 3z = 1$$

$$3y - z = 5$$

Q28.

If  $A =$

$$\begin{vmatrix} 2 & -3 \\ 3 & 2 \\ 1 & 1 \end{vmatrix}$$

Q26.  $2x + y - z = 1$

$$x - y + z = 2$$

$$3x + y - 2z = -1$$

$$\begin{vmatrix} 5 \\ -4 \\ -2 \end{vmatrix} \quad \text{find } A^{-1}$$

Q27.  $x - y = 3$

$$2x + 3y + 4z = 17$$

$$3y - z = 5$$

Using  $A^{-1}$  solve the foll. System of equations

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

## MATRICES

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### ONE MARKS QUESTIONS

1. Show by means of an example that the product of two non-zero matrices can be a zero matrix.
2. Construct a  $3 \times 2$  matrix whose elements are given by  $a_{ij} = e^{ix} \sin jx$ . (Exemplar)
3. Solve for  $x$  and  $y$  for  $x \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$  (Exemplar).
4. Give an example of matrices  $A, B$  and  $C$  such that  $AB = AC$ , Where  $A$  is non-zero matrix, but  $B \neq C$ .
5. Show that  $A^T A$  and  $AA^T$  are both symmetric matrices for any matrix  $A$ . (Exemplar).

### FOUR MARKS QUESTIONS

6. If  $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$  prove that  $A^2 - 4A - 5I = 0$ . Hence find  $A^{-1}$ .
  7. Given  $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$  show by induction that  $A^n = \begin{pmatrix} 1 + 2n & -4n \\ n & 1 - 2n \end{pmatrix}$ .
  8. If  $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$  and  $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$ , Find a matrix  $Z$  such that  $X+Y+Z$  is a zero matrix. (Exemplar).
  9. Find the matrix  $A$  satisfying the matrix equation :  

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
. (Exemplar).
  10. Prove by mathematical induction that  

$$(A^T)^n = (A^n)^T, \text{ where } n \in \mathbb{N} \text{ for any square matrix } A. \text{ (Exemplar).}$$
  11. If  $F(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$  show that  $F(\theta)F(\varphi) = F(\theta + \varphi)$ .
  12. Find the inverse by elementary Operations  $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \\ 3 & 2 & 1 \end{bmatrix}$ .
  13. Express the matrix  $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$  as the sum of a symmetric and skew symmetric matrix.  
(Exemplar).
  14. Find the value of  $x$ , if  

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0.$$
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19TH MARCH 2020

## Determinants Class 12<sup>th</sup>

	Solve for x
Q.1)	Solve $\begin{vmatrix} a + xa - xa - x \\ a - xa + xa - x \\ a - xa + xa + x \end{vmatrix} = 0$
Sol.1)	$c_1 \rightarrow c_1 + c_2 + c_3$ $\Rightarrow \begin{vmatrix} 3a - xa - xa - x \\ 3a - xa + xa - x \\ 3a - xa - xa + x \end{vmatrix} = 0$ taking $(3a - x)$ common from $\Rightarrow (3a - x) \begin{vmatrix} 1a - xa - x \\ 1a + xa - x \\ 1a - xa + x \end{vmatrix} = 0$ $R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$ $\Rightarrow (3a - x) \begin{vmatrix} 1a - xa - x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$ taking $2x$ common from $R_2$ & $R_3$ both $\Rightarrow (3a - x)(2x)(2x) \begin{vmatrix} 1a - xa - x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$ expanding along $R_1$ $\Rightarrow (3a - x)(4x^2)[1(1) - 0 + 0] = 0$ $\Rightarrow (3a - x)(4x^2) = 0$ $\Rightarrow x = 3a, x = 0 \quad \text{ans.}$
Q.2)	Solve $\begin{vmatrix} x - 22x - 33x - 4 \\ x - 42x - 93x - 16 \\ x - 82x - 273x - 64 \end{vmatrix} = 0$
Sol.2)	We have $\begin{vmatrix} x - 22x - 33x - 4 \\ x - 42x - 93x - 16 \\ x - 82x - 273x - 64 \end{vmatrix} = 0$ $R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$ $\Rightarrow \begin{vmatrix} x - 22x - 33x - 4 \\ -2 - 6 - 12 \\ -6 - 24 - 60 \end{vmatrix} = 0$ $c_2 \rightarrow c_2 - 2c_1 \text{ and } c_3 \rightarrow c_3 - 3c_1$ $\Rightarrow \begin{vmatrix} x - 212 \\ -2 - 2 - 6 \\ -6 - 12 - 42 \end{vmatrix} = 0$ taking $(-2)$ and $(-6)$ common from $R_2$ & $R_3$ respectively $\Rightarrow 12 \begin{vmatrix} x - 212 \\ 113 \\ 127 \end{vmatrix} = 0$ expanding along $R_1$ $\Rightarrow 12[(x - 2)(7 - 6) - 1(7 - 3) + 2(2 - 1)] = 0$ $\Rightarrow 12[x - 2 - 4 + 2] = 0$

$$\Rightarrow 12(x - 4) = 0$$

$$\Rightarrow x = 4 \quad \text{ans.}$$

### Proving Questions

Q.3) Show that  $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$

Sol.3) let  $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

$$R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b - a & b^2 - a^2 \\ 0 & c - a & c^2 - a^2 \end{vmatrix}$$

taking (b-a) and (c-a) common from  $R_2$  &  $R_3$  respectively

$$= (b - a)(c - a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b + a \\ 0 & 1 & c + a \end{vmatrix}$$

expanding along  $R_1$

$$= (b - a)(c - a)[1(c + a - b - a) - a(0) + a^2(0)]$$

$$= (b - a)(c - a)(c - b)$$

$$= (a - b)(b - c)(c - a) = \text{RHS} \quad \text{Proved}$$

Q.4) Show that  $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(ab + bc + ca)$

Sol.4) let  $\Delta = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix}$

$$c_1 \rightarrow ac_1; c_2 \rightarrow bc_2 \text{ and } c_3 \rightarrow ac_3$$

$$= \frac{1}{abc} \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ abc & abc & abc \end{vmatrix}$$

taking abc common from  $C_3$

$$= \frac{abc}{abc} \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$c_2 \leftrightarrow c_3$$

$$= - \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$c_1 \leftrightarrow c_2$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$c_2 \rightarrow c_2 - c_1 \text{ and } c_3 \rightarrow c_3 - c_1$$

$$= \begin{vmatrix} 1 & & 0 \\ a^2 & b^2 - a^2 & c^2 - a^2 \\ a^3 & (b - a)(b^2 + ab + a^2) & (c - b)(c^2 + ac + a^2) \end{vmatrix}$$

	<p>taking <math>(b-a)</math> and <math>(c-b)</math> common from <math>c_2</math> &amp; <math>c_3</math></p> $= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a^2 & b+a & c+a \\ a^3 & b^2+ab+a^2 & c^2+ac+a^2 \end{vmatrix}$ <p>expanding along <math>R_1</math></p> $= (b-a)(c-a)[(b+a)(c^2+ac+a^2) - (c+a)(b^2+ab+a^2)]$ $= (b-a)(c-a)[bc^2+abc+a^2b+ac^2+a^2c+a^3 - b^2c-abc-a^2c-ab^2-a^2b-a^3]$ $= (b-a)(c-a)(bc^2+ac^2-b^2c-ab^2)$ $= (b-a)(c-a)[bc(c-b)+a(c^2-b^2)]$ $= (b-a)(c-a)(c-b)[bc+a(c+b)]$ $= (b-a)(c-a)(c-b)(bc+ac+ab)$ $= (a-b)(b-c)(c-a)(ab+bc+ac) = \text{RHS}$
Q.5)	<p>Show that <math>\begin{vmatrix} a+b+2c &amp; a &amp; b \\ c &amp; b+c+2a &amp; b \\ c &amp; a &amp; c+a+2b \end{vmatrix} = 2(a+b+c)^3</math></p>
Sol.5)	<p><math>c_1 \rightarrow c_1 + c_2 + c_3</math></p> $= \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$ <p>taking <math>2(a+b+c)</math> common from <math>C_1</math></p> $= 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$ <p><math>R_2 \rightarrow R_2 - R_1</math> and <math>R_3 \rightarrow R_3 - R_1</math></p> $= 2(a+b+c) \begin{vmatrix} 1 & a & b \\ 0 & b+c+2a & b \\ 0 & a & c+a+2b \end{vmatrix}$ <p>expanding along <math>R_1</math></p> $= 2(a+b+c)[(a+b+c)(a+b+c)] = 2(a+b+c)^3 \quad \text{ans.}$
Q.6)	<p>Show <math>\begin{vmatrix} \alpha &amp; \beta &amp; \gamma \\ \alpha^2 &amp; \beta^2 &amp; \gamma^2 \\ \beta+\gamma &amp; \gamma+\alpha &amp; \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)</math></p>
Sol.6)	<p><math>R_3 \rightarrow R_3 + R_1</math></p> $= \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \alpha+\beta+\gamma & \alpha+\beta+\gamma & \alpha+\beta+\gamma \end{vmatrix}$ <p>taking <math>(\alpha+\beta+\gamma)</math> common from <math>R_3</math></p> $= (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$ <p><math>c_2 \rightarrow c_2 - c_1</math> and <math>c_3 \rightarrow c_3 - c_1</math></p> $= (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \beta-\alpha & \gamma-\alpha \\ \alpha^2 & \beta^2-\alpha^2 & \gamma^2-\alpha^2 \\ 1 & 0 & 0 \end{vmatrix}$ <p>taking <math>(\beta-\alpha)</math> and <math>(\gamma-\alpha)</math> common from <math>C_2</math> &amp; <math>C_3</math> respectively</p>

$$= (\alpha + \beta + \gamma)(\beta - \alpha)(\gamma - \alpha) \begin{vmatrix} \alpha & 1 & 1 \\ \alpha^2 & \beta - \alpha & \gamma - \alpha \\ 1 & 0 & 0 \end{vmatrix}$$

expanding

$$\begin{aligned} &= (\alpha + \beta + \gamma)(\beta - \alpha)(\gamma - \alpha)[\alpha(0) + 1(\gamma + \alpha) + 1(-\beta - \alpha)] \\ &= (\alpha + \beta + \gamma)(\beta - \alpha)(\gamma - \alpha)(\gamma + \alpha - \beta - \alpha) \\ &= (\alpha + \beta + \gamma)(\beta - \alpha)(\gamma - \alpha)(\gamma - \beta) \\ &= (\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)(\alpha + \beta + \gamma) = \text{RHS} \end{aligned}$$

Q.7)

Show  $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

Sol.7)

$$\begin{aligned} R_1 &\rightarrow R_1 + R_2 + R_3 \\ &= \begin{vmatrix} 2(a+b+c) & 0 & a+b+c \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} \end{aligned}$$

taking  $(a+b+c)$  common from  $R_1$

$$= (a+b+c) \begin{vmatrix} 2 & 0 & 1 \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

$$c_1 \rightarrow c_1 - 2c_3$$

$$= (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ c+a-2b & b-c & b \\ a+b-2c & c-a & c \end{vmatrix}$$

expanding along  $R_1$

$$\begin{aligned} &= (a+b+c)[1(c+a-2b)(c-a) - (a+b-2c)(b-c)] \\ &= (a+b+c)[c^2 - ac + ac - a^2 - 2bc + 2ab - ab + ac - b^2 + bc + 2bc - 2c^2] \\ &= (a+b+c)(-a^2 - b^2 - c^2 + ab + bc + ca) \\ &= -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) \\ &= -[a^3 + b^3 + c^3 - 3abc] \\ &= 3abc - a^3 - b^3 - c^3 = \text{RHS} \quad \text{ans.} \end{aligned}$$

Q.8)

Show  $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (x^3 - 1)^2$

Sol.8)

$$\begin{aligned} c_1 &\rightarrow c_1 + c_2 + c_3 \\ &= \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix} \end{aligned}$$

taking  $(1+x+x^2)$  common from  $C_1$

$$= (1+x+x^2) \begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$$

$$R \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$$

$$= (1+x+x^2) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1-x & x-x^2 \\ 0 & x^2-x & 1-x^2 \end{vmatrix}$$

	$= (1 + x + x^2) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 - x & x(1 - x) \\ 0 & -x(1 - x) & (1 + x)(1 - x) \end{vmatrix}$ <p>taking <math>(1 - x)</math> common from <math>R_2</math> &amp; <math>R_3</math> both</p> $= (1 + x + x^2)(1 - x)^2 \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & x \\ 0 & -x & 1 + x \end{vmatrix}$ <p>expanding</p> $= (1 + x + x^2)(1 - x)^2 [1 + x + x^2]$ $= (1 - x)^2 (1 + x + x^2)^2$ $= [(1 - x)(1 + x + x^2)]^2$ $= (1 - x^3)^2 \quad \dots \{a^3 - b^3 = (a - b)(a^2 + ab + b^2)\}$ $= (x^3 - 1)^2 = \text{RHS} \quad \dots \{(a - b)^2 = (b - a)^2\}$
Q.9)	<p>Show <math>\begin{vmatrix} a^2 &amp; 2ab &amp; b^2 \\ b^2 &amp; a^2 &amp; 2ab \\ 2ab &amp; b^2 &amp; b^2 \end{vmatrix} = (a^3 + b^3)^2</math></p>
Sol.9)	$c_1 \rightarrow c_1 + c_2 + c_3$ $= \begin{vmatrix} a^2 + 2ab + b^2 & 2ab & b^2 \\ a^2 + 2ab + b^2 & a^2 & 2ab \\ a^2 + 2ab + b^2 & b^2 & a^2 \end{vmatrix}$ $= (a + b)^2 \begin{vmatrix} 1 & 2ab & b^2 \\ 1 & a^2 & 2ab \\ 1 & b^2 & a^2 \end{vmatrix}$ $R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$ $= (a + b)^2 \begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2 - 2ab & 2ab - b^2 \\ 0 & b^2 - 2ab & a^2 - b^2 \end{vmatrix}$ <p>expanding along <math>R_1</math></p> $= (a + b)^2 [(a^2 - 2ab)(a^2 - b^2) - (b^2 - 2ab)(2ab - b^2)]$ $= (a + b)^2 [a^4 - a^2b^2 - 2a^3b + 2ab^3 - 2ab^3 + b^4 + 2a^2b^2 - 2ab^3]$ $= (a + b)^2 [a^4 + b^4 + a^2b^2 - 2a^3b + 2a^2b^2 - 2ab^3]$ $= (a + b)^2 (a^2 - ab + b^2)^2 \quad \dots \{(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca\}$ $= [(a + b)(a^2 - ab + b^2)]^2$ $= (a^3 + b^3)^2 \quad \text{ans.}$
Q.10)	<p>Show <math>\begin{vmatrix} a^2 + 1abac \\ abb^2 + 1bc \\ cacbc^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2</math></p>
Sol.10)	$R_1 \rightarrow aR_1, R_2 \rightarrow bR_2 \text{ and } R_3 \rightarrow cR_3$ $= \frac{1}{abc} \begin{vmatrix} a^3 + aa^2ba^2c \\ ab^2b^3 + bb^2c \\ c^2ac^2bc^3 + c \end{vmatrix}$ <p>taking <math>a, b, c</math> common from <math>c_1, c_2</math> &amp; <math>c_3</math> respectively</p> $= \frac{abc}{abc} \begin{vmatrix} a^2 + 1a^2a^2 \\ b^2b^2 + 1b^2 \\ c^2c^2c^2 + 1 \end{vmatrix}$ $R_1 \rightarrow R_1 + R_2 + R_3$



$$= \begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2b^2+1b^2 & & \\ c^2c^2c^2+1 & & \end{vmatrix}$$

$$= (1+a^2+b^2+c^2) \begin{vmatrix} 1 & 1 & 1 \\ b^2b^2+1b^2 & & \\ c^2c^2c^2+1 & & \end{vmatrix}$$

$$c_2 \rightarrow c_2 - c_1 \text{ and } c_3 \rightarrow c_3 - c_1$$

$$= (1+a^2+b^2+c^2) \begin{vmatrix} 1 & 0 & 0 \\ b^2b^2+1b^2 & & \\ c^2c^2c^2+1 & & \end{vmatrix}$$

expanding

$$= (1+a^2+b^2+c^2)[1] = 1+a^2+b^2+c^2 = \text{RHS}$$

## Determinants

### Class 12<sup>th</sup>

Q.1)	Show that $\begin{vmatrix} 1 & 1 & 1 \\ mc_1 & m+1c_1 & m+2c_1 \\ mc_2 & m+1c_2 & m+2c_2 \end{vmatrix} = 1$
Sol.1)	<p>We have <math display="block">\begin{vmatrix} 1 &amp; 1 &amp; 1 \\ mc_1 &amp; m+1c_1 &amp; m+2c_1 \\ mc_2 &amp; m+1c_2 &amp; m+2c_2 \end{vmatrix}</math></p> $= \begin{vmatrix} 1 & 1 & 1 \\ \frac{m(m-1)}{2} & \frac{(m+1)m}{m} & \frac{(m+2)(m+1)}{2} \end{vmatrix} \dots \left\{ \begin{array}{l} nc_1 = n \\ nc_2 = \frac{n(n-1)}{2} \end{array} \right\}$ <p>taking <math>\left(\frac{1}{2}\right)</math> common from <math>R_3</math></p> $= \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ m & m+1 & m+2 \\ m^2 - m & m^2 + m & m^2 + 3m + 2 \end{vmatrix}$ <p><math>c_2 \rightarrow c_2 - c_1</math> and <math>c_3 \rightarrow c_3 - c_2</math></p> $= \frac{1}{2} \begin{vmatrix} 1 & 0 & 0 \\ m & 1 & 2 \\ m^2 - m & 2m & 4m + 2 \end{vmatrix}$ <p>expanding along <math>R_1</math></p> $= \frac{1}{2} [4m + 2 - 4m]$ $= \frac{2}{2} = 1 = \text{RHS} \quad \text{ans.}$
Q.2)	Show $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$
Sol.2)	<p>We have <math display="block">\begin{vmatrix} (b+c)^2 &amp; ba &amp; ca \\ ab &amp; (c+a)^2 &amp; cb \\ ac &amp; bc &amp; (a+b)^2 \end{vmatrix}</math></p> $= R_1 \rightarrow aR_1, R_2 \rightarrow bR_2, R_3 \rightarrow cR_3$ $= \frac{1}{abc} \begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & b(c+a)^2 & cb^2 \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$ <p>taking <math>a, b, c</math> common from <math>c_1, c_2</math> and <math>c_3</math> resp.</p> $= \frac{1}{abc} \begin{vmatrix} a(b+c)^2 & ba^2 & ca^2 \\ ab^2 & b(c+a)^2 & cb^2 \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix}$ <p><math>c_1 \rightarrow c_1 - c_3</math> and <math>c_2 \rightarrow c_2 - c_3</math></p> $= \begin{vmatrix} (b+c+a)(b+c-a) & 0 & a^2 \\ 0 & (c+a+b)(c-a-b) & b^2 \\ (c+a+b)(c-a-b) & (c+a+b)(c-a-b) & (a+b)^2 \end{vmatrix}$ <p>taking <math>(a+b+c)</math> common from <math>C_1</math> &amp; <math>C_2</math> both</p>

	$= (a + b + c)^2 \begin{vmatrix} b + c - a & 0 & a^2 \\ 0 & c + a - b & b^2 \\ c - a - b & c - a - b & (a + b)^2 \end{vmatrix}$ $R_3 \rightarrow R_3(R_1 + R_2)$ $= (a + b + c)^2 \begin{vmatrix} b + c - a & 0 & a^2 \\ 0 & c + a - b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$ $c_1 \rightarrow ac_1 \text{ and } c_2 \rightarrow bc_2$ $= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab + ac - a^2 & 0 & a^2 \\ 0 & bc + ab - b^2 & b^2 \\ -2ab & -2ab & 2ab \end{vmatrix}$ $c_1 \rightarrow c_1 + c_3 \text{ and } c_2 \rightarrow c_2 + c_3$ $= \frac{(a+b+c)^2}{ab} \begin{vmatrix} ab + ac & a^2 & a^2 \\ b^2 & bc + ab & b^2 \\ 0 & 0 & 2ab \end{vmatrix}$ <p>taking a, b and 2ab common from R<sub>1</sub>, R<sub>2</sub> and R<sub>3</sub> resp.</p> $= \frac{(a+b+c)^2}{ab} \cdot ab(2ab) \begin{vmatrix} b + c & a & a \\ b & c + a & b \\ 0 & 0 & 1 \end{vmatrix}$ <p>expanding</p> $= 2ab(a + b + c)^2[(b + c)(c + a) - a(b) + a(0)]$ $= 2ab(a + b + c)^2(bc + ab + c^2 + ac - ab)$ $= 2ab(a + b + c)^2 \cdot c(b + c + a)$ $= 2abc(a + b + c)^3 = \text{RHS} \quad \text{ans.}$
Q.3)	<p>Show that <math>\begin{vmatrix} 1 + a^2 - b^2 &amp; 2ab &amp; -2b \\ 2ab &amp; 1 - a^2 + b^2 &amp; 2a \\ 2b &amp; -2a &amp; 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3</math></p>
Sol.3)	<p>We have <math>\begin{vmatrix} 1 + a^2 - b^2 &amp; 2ab &amp; -2b \\ 2ab &amp; 1 - a^2 + b^2 &amp; 2a \\ 2b &amp; -2a &amp; 1 - a^2 - b^2 \end{vmatrix}</math></p> <p>Main step <math>c_1 \rightarrow c_1 - bc_3</math> and <math>c_2 \rightarrow ac_3</math></p> $= \begin{vmatrix} 1 + a^2 - b^2 + 2b^2 & 2ab - 2ab & -2b \\ 2ab - 2ab & 1 - a^2 + b^2 + 2a^2 & 2a \\ 2b - b + a^2b + b^3 & -2a + a - a^3 - ab^2 & 1 - a^2 - b^2 \end{vmatrix}$ $= \begin{vmatrix} 1 + a^2 + b^2 & 0 & -2b \\ 0 & 1 + a^2 + b^2 & 2a \\ b(1 + a^2 + b^2) & -a(1 + a^2 + b^2) & 1 - a^2 - b^2 \end{vmatrix}$ <p>taking <math>(1 + a^2 + b^2)</math> common from <math>c_1</math> and <math>c_2</math></p> $= (1 + a^2 + b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1 - a^2 - b^2 \end{vmatrix}$ <p>expanding</p> $= (1 + a^2 + b^2)^2 [1[1 - a^2 - b^2 + 2a^2] - 2b(-b)]$ $= (1 + a^2 + b^2)^2 [1 - a^2 - b^2 + 2a^2 + 2b^2]$ $= (1 + a^2 + b^2)^2 (1 + a^2 + b^2)$ $= (1 + a^2 + b^2)^3 = \text{RHS} \quad \text{ans.}$

## Solving System of Linear Equations (Matrix Method)

Q.4) Solve the equations using matrix method  $x + 2y + z = 7$  ;  $x + 3z = 11$  ;  $2x - 3y = 1$

Sol.4) The given equation are

$$x + 2y + z = 7$$

$$x + 0y + 3z = 11$$

$$2x - 3y + 0z = 1$$

these equation can be written in matrix form

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

(or)  $Ax = B$

$$\Rightarrow x = A^{-1}B$$

$$\text{Where } A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix} ; B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix} \text{ \& } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Now

$$|A| = 1(0 + 9) - 2(0 - 6) + 1(-3 - 0) = 9 + 12 - 3$$

$$|A| = 18 \neq 0$$

$\therefore$  system is consistent and unique solution

Cofactors

$$c_{11} = 9 ; c_{12} = -6 ; c_{14} = -3$$

$$c_{21} = -3 ; c_{22} = -2 ; c_{23} = 7$$

$$c_{31} = 6 ; c_{32} = -2 ; c_{33} = -2$$

$$\text{Now } Adj(A) = \begin{bmatrix} 9 & 6 & -3 \\ -3 & -2 & 7 \\ 6 & -2 & -2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot Adj(A)$$

$$A^{-1} = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$$

We have  $x = A^{-1}B$

$$x = \frac{1}{18} \begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{18} \begin{bmatrix} 63 - 33 + 6 \\ 42 - 22 - 2 \\ -21 + 77 - 2 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{18} \begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\therefore x = 2$  ,  $y = 1$  ,  $z = 3$  is the required solution ans.

Q.5) Solve the equations

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10 ; \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10 ; \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

Sol.5) The given equations are

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$$

$$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

These equations can be written in matrix form

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

(or)  $AX = B$

$$\Rightarrow X = A^{-1}B$$

$$\text{Where } A = \begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}; B = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}; X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$$

$$|A| = 2(2 + 1) + 3(2 - 3) + 3(-1 - 3) = 6 - 3 - 12 = -9$$

$$|A| = -9 \neq 0 \quad \therefore \text{system is consistent and unique solution}$$

Cofactors

$$c_{11} = 3 \quad c_{12} = 1 \quad c_{13} = -4$$

$$c_{21} = 3 \quad c_{22} = -5 \quad c_{23} = -7$$

$$c_{31} = -6 \quad c_{32} = 1 \quad c_{33} = 5$$

$$\therefore \text{Adj}A = \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}A = -\frac{1}{9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix}$$

We have  $X = A^{-1}B$

$$X = -\frac{1}{9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

$$X = -\frac{1}{9} \begin{bmatrix} 30 + 30 - 78 \\ 10 - 50 + 13 \\ -40 - 70 + 65 \end{bmatrix}$$

$$\begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = -\frac{1}{9} \begin{bmatrix} -18 \\ -27 \\ -45 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{2}; y = \frac{1}{3} \text{ and } z = \frac{1}{5} \text{ is the required solution} \quad \text{Ans.....}$$

Q.6)

$$\text{Find } A^{-1}, \text{ where } A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}. \text{ Hence solve the system of equations } x +$$

$$2y - 3z = -4,$$

$$2x + 3y + 2z = 2 \text{ and } 3x - 3y - 4z = 11$$

Sol.6)

$$\text{We have, } A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$$

$$|A| = 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9) = -6 + 28 + 45$$

$$|A| = 67 \neq 0 \quad \therefore (A \text{ is Invertible } | \text{consistent} | \text{ unique solution})$$

Cofactors

$$c_{11} = -6 \quad c_{12} = 14 \quad c_{13} = -15$$

$$c_{21} = 17 \quad c_{22} = 5 \quad c_{23} = 9$$

$$c_{31} = 13 \quad c_{32} = -8 \quad c_{33} = -1$$

$$\therefore \text{Adj}(A) = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$\text{Now } A^{-1} = \frac{1}{|A|} \cdot \text{Adj}A$$

$$A^{-1} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \quad \dots\dots(1)$$

Given equation are

$$x + y - 3z = -4$$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 1$$

These equation can be written in the form

$$\begin{bmatrix} 1 & 1 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$\text{(or) } AX = B \Rightarrow X = A^{-1}B$$

$$\text{Where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$X = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix} \quad \dots\dots\{A^{-1} \text{ from eq. (i)}\}$$

$$X = \frac{1}{67} \begin{bmatrix} 24 + 34 + 143 \\ -56 + 10 - 88 \\ 60 + 18 - 11 \end{bmatrix} = \frac{1}{67} \begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore x = 3, y = -2, z = 1$  is the required solution ans.

Q.7)

$$\text{If } A = \begin{bmatrix} 1 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}. \text{ Find } A^{-1} \text{ and hence solve the equation } x + 2y + z = 4 ; -x +$$

$$y + z = 0 \text{ and } x - 3y + z = 2.$$

Sol.7)

$$\text{We have } A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

Do yourself

$$|A| = 10 \neq 0 \quad \therefore (A \text{ is invertible})$$

$$\text{Adj}A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}A$$

$$A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \quad \dots\dots(1)$$

Given equation are

$$x + 2y + z = 4$$

$$-x + y + z = 0$$

$$x - 3y + z = 2$$

$\rightarrow$  the matrix of above equation is clearly the transpose of given matrix A

$\therefore$  these equations can be written in the form

$$A'X = B \quad \text{where } B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow X = (A^{-1})^{-1}B$$

$$\Rightarrow X = (A^{-1})^{-1}B \quad \dots\dots\dots\{\text{By prop. } (A^{-1})^{-1} = (A^{-1})^1\}$$

$$\Rightarrow X = \frac{1}{10} \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{10} \begin{bmatrix} 16+2 \\ 8-4 \\ 8+6 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 18 \\ 4 \\ 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9/5 \\ 2/5 \\ 7/5 \end{bmatrix}$$

$$\Rightarrow x = \frac{9}{5}, y = \frac{2}{5}, z = \frac{7}{5} \text{ is the req. solution} \quad \text{ans..}$$

Q.8)

Determine the product  $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$  and hence (or) use it to solve the

equations  $x - y + z = 4$ ;  $x - 2y - 2z = 9$ ;  $2x + y + 3z = 1$

Sol.8)

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$$CA = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$\Rightarrow CA = 8I$$

Post multiply by  $A^{-1}$

$$\Rightarrow CAA^{-1} = 8IA^{-1}$$

$$\Rightarrow CI = 8A^{-1} \quad \dots\dots\dots \left\{ \begin{array}{l} AA^{-1} = I \\ IA^{-1} = A^{-1} \end{array} \right\}$$

$$\Rightarrow A^{-1} = \frac{1}{8}C = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

Given equation are

$$x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1$$

There equation can be in the form

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

(or)  $AX = B$

$$\Rightarrow X = A^{-1}B \quad \text{where } B = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{8} \begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$\Rightarrow x = 3, y = -2 \text{ and } z = -1 \text{ is the req. solution} \quad \text{ans.}$$

Q.9)

$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$  find  $AB$  and hence solve the equations

$$x - 2y = 0; 2x + y + 3z = 8; -2y + x = 7$$

Sol.9)

$$AB = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$\Rightarrow AB = 11I$$

Pre by  $A^{-1}$

$$\Rightarrow A^{-1}AB = 11A^{-1}I$$

$$\Rightarrow IB = 11A^{-1}$$

$$\Rightarrow B = 11A^{-1}$$

$$\Rightarrow A^{-1} = \frac{1}{11}B = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

Given equations are

$$x - 2y = 10$$

$$2x + y + 3z = 7$$

$$-2y + 0y + z = 7$$

These equations can be written in the form

$$AX = C \Rightarrow X = A^{-1}C$$

$$\text{Where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } C = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$X = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{11} \begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

$\therefore x = 4, y = -3, z = 1$  is the required solution ans.

Q.10) Show that system of equations is consistent and also find the solution

$$2x - y + 3z = 5; 3x + 2y - z = 7; 4x + 5y - 5z = 9$$

Sol.10) Given equation are

$$2x - y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y - 5z = 9$$

given equation can be written in the form

$$AX = B \Rightarrow X = A^{-1}B$$

$$\text{where } A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & -5 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ \& } B = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

$|A| = 0$  {solution can be infinite many or no solution}

$$\text{Now } AdjA = \begin{bmatrix} -5 & 10 & -5 \\ 11 & -22 & 11 \\ 7 & -14 & 7 \end{bmatrix}$$

$$\text{Now } (AdjA)B = \begin{bmatrix} -5 & 10 & -5 \\ 11 & -12 & 11 \\ 7 & -14 & 7 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0$$

Since  $|A| = 0$  also  $(AdjA) \cdot B = 0$

$\therefore$  System is consistent and Infinite many solutions

$\rightarrow$  Put  $z = k$  in first two equations, we get

$$2x - y = 5 - 3k \quad \dots (k \in R)$$

$$3x + 2y = 7 + k$$

$$\text{(or)} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 - 3k \\ 7 + k \end{bmatrix}$$



$$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \end{bmatrix} ; B = \begin{bmatrix} 5 - 3k \\ 7 + k \end{bmatrix}$$

$$|A| = 7 \text{ and } AdjA = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 5 - 3k \\ 7 + k \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 17 - 5k \\ 11k - 1 \end{bmatrix}$$

$$\therefore x = \frac{17-5k}{7} ; y = \frac{11k-1}{7} \text{ and } z = k \quad \text{ans.}$$

## Determinants

### Class 12<sup>th</sup>

Q.1)	Show $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$
Sol.1)	$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ $C_3 \rightarrow C_3 + C_2$ $= \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$ <p>taking (a + b + c) common from C<sub>3</sub></p> $= (a + b + c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$ $= (a + b + c) \times 0 = 0 \quad \dots\{\because C_1 \& C_3 \text{ are identical}\}$
Q.2)	Show that $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0$
Sol.2)	$\text{let } \Delta = \begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix}$ $c_1 \rightarrow c_1 + c_2 + c_3$ $= \begin{vmatrix} 0 & c-a & a-b \\ 0 & a-b & b-c \\ 0 & b-c & c-a \end{vmatrix}$ $= 0 \quad \dots\{\text{all elements of } C_1 \text{ are } z \text{ easily}\}$
Q.3)	Show that $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+S) \\ \sin\beta & \cos\beta & \sin(\beta+S) \\ \sin\gamma & \cos\gamma & \sin(\gamma+S) \end{vmatrix} = 0$
Sol.3)	$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha \cdot \cos S + \cos\alpha \cdot \sin S \\ \sin\beta & \cos\beta & \sin\beta \cdot \cos S + \cos\beta \cdot \sin S \\ \sin\gamma & \cos\gamma & \sin\gamma \cdot \cos S + \cos\gamma \cdot \sin S \end{vmatrix}$ <p>Applying sum property in C<sub>3</sub></p> $= \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha \cdot \cos S \\ \sin\beta & \cos\beta & \sin\beta \cdot \cos S \\ \sin\gamma & \cos\gamma & \sin\gamma \cdot \cos S \end{vmatrix} + \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha \cdot \sin S \\ \sin\beta & \cos\beta & \cos\beta \cdot \sin S \\ \sin\gamma & \cos\gamma & \cos\gamma \cdot \sin S \end{vmatrix}$ $= \cos S \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha \\ \sin\beta & \cos\beta & \sin\beta \\ \sin\gamma & \cos\gamma & \sin\gamma \end{vmatrix} + \sin S \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha \\ \sin\beta & \cos\beta & \cos\beta \\ \sin\gamma & \cos\gamma & \cos\gamma \end{vmatrix}$ $= \cos S(0) + \sin S(0) = 0 \quad \text{ans.}$

Q.4)	If a, b, c are in A.P find value of $\begin{vmatrix} 2y + 45y + 78y + a \\ 3y + 56y + 89y + b \\ 4y + 67y + 910y + c \end{vmatrix}$
Sol.4)	<p>let <math>\Delta = \begin{vmatrix} 2y + 45y + 78y + a \\ 3y + 56y + 89y + b \\ 4y + 67y + 910y + c \end{vmatrix}</math></p> <p>given a, b, c are in A.P <math>a + c = 2b</math></p> <p><math>\therefore R_1 \rightarrow R_1 + R_3</math></p> $= \begin{vmatrix} 6y + 1012y + 1618y + a + c \\ 3y + 56y + 89y + b \\ 4y + 67y + 910y + c \end{vmatrix}$ $= \begin{vmatrix} 6y + 1012y + 1618y + 2b \\ 3y + 56y + 89y + b \\ 4y + 67y + 910y + c \end{vmatrix} \quad \dots\{\because a + c = 2b\}$ <p>taking 2 common from <math>R_1</math></p> $= 2 \begin{vmatrix} 3y + 56y + 89y + b \\ 3y + 56y + 89y + b \\ 4y + 67y + 910y + c \end{vmatrix}$ <p>clearly <math>R_1</math> and <math>R_2</math> are identical</p> <p><math>\therefore 2 \times 0 = 0</math>                      ans.</p>
Q.5)	Show that $\begin{vmatrix} (a^x + a^{-x})^2(a^x - a^{-x})^2 1 \\ (a^y + a^{-y})^2(a^y - a^{-y})^2 1 \\ (a^z + a^{-z})^2(a^z - a^{-z})^2 1 \end{vmatrix} = 0$
Sol.5)	<p><math>C_1 \rightarrow C_1 - C_2</math></p> $= \begin{vmatrix} (a^x + a^{-x})^2 - (a^x - a^{-x})^2(a^x - a^{-x})^2 1 \\ (a^y + a^{-y})^2 - (a^y - a^{-y})^2(a^y - a^{-y})^2 1 \\ (a^z + a^{-z})^2 - (a^z - a^{-z})^2(a^z - a^{-z})^2 1 \end{vmatrix}$ $= \begin{vmatrix} 4(a^x - a^{-x})^2 1 \\ 4(a^y - a^{-y})^2 1 \\ 4(a^z - a^{-z})^2 1 \end{vmatrix} \quad \dots\{(a + b)^2 - (a - b)^2 = 4ab\}$ <p>4}</p> $= 4 \begin{vmatrix} 1(a^x - a^{-x})^2 1 \\ 1(a^y - a^{-y})^2 1 \\ 1(a^z - a^{-z})^2 1 \end{vmatrix}$ $= 4 \times 0 = 0 \quad \dots\{C_1 \& C_3 \text{ are identical}\}$
Q.6)	Show that $\begin{vmatrix} 4115 \\ 7979 \\ 2953 \end{vmatrix} = 0$
Sol.6)	let $\Delta = \begin{vmatrix} 4115 \\ 7979 \\ 2953 \end{vmatrix}$

	$c_2 \rightarrow c_2 + 8c_3$ $= \begin{vmatrix} 41415 \\ 79799 \\ 29293 \end{vmatrix} = 0 \quad \dots \{C_1 \& C_3 \text{ are identical}\}$
Q.7)	Show $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & a+b \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$
Sol.7)	$R_1 \rightarrow aR_1, R_2 \rightarrow bR_2 \text{ and } R_3 \rightarrow cR_3$ $= \frac{1}{abc} \begin{vmatrix} ab^2c^2 & abc & ab+ac \\ bc^2a^2 & abc & bc+ab \\ ca^2b^2 & abc & ca+bc \end{vmatrix}$ <p>taking abc common from <math>C_1</math> and <math>C_2</math></p> $= \frac{1}{abc} \cdot (abc)(abc) \begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+bc \end{vmatrix}$ $c_3 \rightarrow c_3 + c_1$ $= abc \begin{vmatrix} bc & 1 & ab+bc+ca \\ ca & 1 & ab+bc+ca \\ ab & 1 & ab+bc+ca \end{vmatrix}$ <p>taking <math>(ab+bc+ca)</math> common from <math>C_3</math></p> $= abc(ab+bc+ca) \begin{vmatrix} bc & 1 & 1 \\ ca & 1 & 1 \\ ab & 1 & 1 \end{vmatrix}$ $= abc(ab+bc+ca)(0) = 0 \quad \dots \dots \{C_1 \& C_3 \text{ are identical}\}$
Q.8)	Show that $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$
Sol.8)	$\text{let } \Delta = \begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix}$ <p>Applying sum property in <math>C_3</math></p> $= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ $R_1 \rightarrow aR_1, R_2 \rightarrow bR_2, R_3 \rightarrow cR_3$ $= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^3 \end{vmatrix} - \frac{1}{abc} \begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$ <p>taking abc common from <math>C_3</math> in 2<sup>nd</sup> Det.</p> $= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{abc}{abc} \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$ $c_2 \leftrightarrow c_3$

	$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & b^2 \end{vmatrix}$ <p>again <math>c_1 \leftrightarrow c_2</math></p> $= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ $= 0 \quad \text{ans.}$
Q.9)	If a , b , c are the $P^{th}$ , $q^{th}$ and $r^{th}$ terms of G.P then show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$
Sol.9)	<p>We know <math>n^{th}</math> term of G.P : <math>a_n = ar^{n-1}</math>  here let <math>A \rightarrow I^{st}</math> term and R common ratio  <math>\therefore a_p = a = AR^{p-1}</math>; <math>a_q = b = AR^{q-1}</math>; <math>a_r = c = AR^{r-1}</math>  taking log on both sides  <math>\log a = \log(AR^{p-1})</math>; <math>\log b = \log(AR^{q-1})</math>; <math>\log c = \log(AR^{r-1})</math>  <math>\Rightarrow \log a = \log A + (p-1)\log R</math>  <math>\log b = \log A + (q-1)\log R</math>  <math>\log c = \log A + (r-1)\log R</math></p> <p>Now let <math>\Delta = \begin{vmatrix} \log a &amp; p &amp; 1 \\ \log b &amp; q &amp; 1 \\ \log c &amp; r &amp; 1 \end{vmatrix}</math></p> <p>putting values of <math>\log a</math>, <math>\log b</math> and <math>\log c</math></p> $= \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$ <p>Applying sum property in <math>c_1</math></p> $= \begin{vmatrix} \log A & p & 1 \\ \log A & q & 1 \\ \log A & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix}$ $= \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & p & 1 \\ q-1 & q & 1 \\ r-1 & r & 1 \end{vmatrix}$ <p><math>c_1 \rightarrow c_1 + c_3</math></p> $= \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p & p & 1 \\ q & q & 1 \\ r & r & 1 \end{vmatrix}$ $= \log A \times (0) + \log R \times (0)$ $= 0 + 0 = 0 \quad \text{ans.}$
Q.10)	Show $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$

Sol.10)

$$\begin{aligned} \text{let } \Delta &= \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} \\ &= \begin{vmatrix} a & x & p \\ b & y & q \\ c & z & r \end{vmatrix} \quad \dots \{ \cdot, |A| = |A'| \} \end{aligned}$$

$$c_1 \leftrightarrow c_2$$

$$= - \begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$$

$$R_2 \leftrightarrow R_1$$

$$= (-)(-) \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$$

$$= \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix} = \text{RHS} \quad \text{Proved}$$

# Determinants

## Class 12<sup>th</sup>

### Short Questions

Q.1) Order  $3 \times 3$  find  $|A^{-1}| = ?$

Sol.1) We have  $|A^{-1}| = \left| \frac{1}{|A|} \cdot \text{Adj } A \right|$   

$$= \frac{1}{|A|^3} |\text{Adj } A|$$
  

$$= \frac{1}{|A|^3} \cdot |A|^{3-1} = \frac{1}{|A|^3} \cdot |A|^2$$
  

$$\therefore |A^{-1}| = \frac{1}{|A|} \quad \text{Ans.}$$

Q.2) Order  $3 \times 3$  ;  $|A| = 3$  and  $|2AB| = 120$  find  $|B'| = ?$

Sol.2) We have  $|2AB| = 2^3 |AB|$   

$$120 = 2^3 |A| |B|$$
  

$$120 = 8 \times 3 \times |B|$$
  

$$5 = |B|$$
  
 Since  $|B'| = |B|$   

$$\Rightarrow |B'| = 5 \quad \text{Ans.....}$$

Q.3) Order  $2 \times 2$  ;  $\text{Adj } A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$  and  $\text{Adj } B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$  find  $\text{Adj}(AB) = ?$

Sol.3) We have  $\text{Adj}(AB) = (\text{Adj } B) (\text{Adj } A)$   

$$= \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 21 & 17 \end{bmatrix} \quad \text{Ans....}$$

Q.4). Order  $2 \times 2$  ;  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  find  $A(\text{Adj } A)$  without finding  $\text{Adj } A$ .

Sol.4) We have,  $A|\text{Adj } A| = |A| I$   

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
  

$$= (-2) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
  

$$A(\text{Adj } A) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \quad \text{Ans....}$$

Q.5). If  $n = 3 \times 3$  find  $|\text{Adj}(\text{Adj } A)|$  and  $|A| = 5$

Sol.5) We have  $|\text{Adj}(\text{Adj } A)| = |A|^{n-1}$   

$$= |\text{Adj } A|^2$$
  

$$= (|A|^{3-1})^2 = |A|^4$$
  

$$= (5)^4 = 625 \quad \text{Ans.}$$

Q.6) If  $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$  find  $(BA)^{-1}$  and  $B^{-1} \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

Sol.6) We know  $(BA)^{-1} = A^{-1} B^{-1}$   

$$|B| = 12 - 12 = 0 \quad \Rightarrow \quad B \text{ is non invertible}$$
  

$$\therefore (BA)^{-1} \text{ not possible}$$

Q7). If  $-1 \leq x < 0$  ;  $0 \leq y < 1$  and  $1 \leq z < 2$

$$\text{Find } \Delta = \begin{vmatrix} [x] + 1 & [y] & [z] \\ [x] & [y] + 1 & [z] \\ [x] & [y] & [z] + 1 \end{vmatrix}$$

Sol.7) Since  $-1 \leq x < 0 \quad \therefore [x] = -1$

$$0 \leq y < 1 \quad \therefore [y] = 0$$

$$1 \leq z < 2 \quad \therefore [z] = 1$$

$$\therefore \Delta = \begin{bmatrix} -1+1 & 0 & 1 \\ -1 & 0+1 & 1 \\ -1 & 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix} = 1 \quad \text{Ans.....}$$

Q.8).

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 5 & 5 \\ 3 & -1 & 4 \end{bmatrix} \text{ find } M_{32} \text{ and } C_{23}$$

Sol.8)

$$M_{32} = 10 - 12 = -12$$

$$C_{23} = 8 - 9 = (-1) = +1 \quad \text{ans.}$$

(sign change)



## Determinants

### Class 12<sup>th</sup>

Q.1)	Show that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab$
Sol.1)	We have $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ taking a, b, c common from $R_1, R_2$ & $R_3$ respectively $= abc \begin{vmatrix} \frac{1}{a} + 1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b} + 1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c} + 1 \end{vmatrix}$ $R_1 \rightarrow R_1 + R_2 + R_3$ $= abc \begin{vmatrix} 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} & 1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \\ \frac{1}{b} & \frac{1}{b} + 1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c} + 1 \end{vmatrix}$ $= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b} + 1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c} + 1 \end{vmatrix}$ $c_2 \rightarrow c_2 - c_1 \text{ and } c_3 \rightarrow c_3 - c_1$ $= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$ expanding along $R_1$ $= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \times 1$ $= abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = \text{RHS}$ $= abc + bc + ca + ab = \text{RHS} \quad \text{ans.}$
Q.2)	Show $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$
Sol.2)	We have $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix}$ taking a common from $C_1$ $= \begin{vmatrix} 1 & a+b & a+b+c \\ 2 & 3a+2b & 4a+3b+2c \\ 3 & 6a+3b & 10a+6b+3c \end{vmatrix}$ $R_2 \rightarrow R_2 - 2R_1 \text{ and } R_3 \rightarrow R_3 - 3R_1$

$$= a \begin{vmatrix} 1 & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 3a & 7a+3b \end{vmatrix}$$

$R_3 \rightarrow R_3 - 3R_2$

$$= a \begin{vmatrix} 1 & a+b & a+b+c \\ 0 & a & 2a+b \\ 0 & 0 & a \end{vmatrix}$$

expanding along  $R_1$

$$= a[a^2] = a^3 \quad \text{ans.}$$

Q.3) If  $x, y, z$  are different and  $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$  then show  $xyz = -1$ .

Sol.3) We have  $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$

applying sum property in  $C_3$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$$

taking  $x, y, z$  common  $R_1, R_2, R_3$  resp.

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + xyz \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = 0$$

$c_2 \leftrightarrow c_3$

$$\Rightarrow - \begin{vmatrix} x & 1 & x^2 \\ y & 1 & y^2 \\ z & 1 & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$c_1 \leftrightarrow c_2$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} (1 + xyz) = 0$$

$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & z-x & z^2-x^2 \end{vmatrix} (1 + xyz) = 0$$

$$\Rightarrow (y-x)(z-x) \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} (1 + xyz) = 0$$

expanding along  $R_1$

$$\Rightarrow (y-z)(z-x)[z+x-y-x](1 + xyz) = 0$$

$$\Rightarrow (y-x)(z-x)(z-y)(1 + xyz) = 0$$

but  $\Rightarrow y-x \neq 0$   
 $z-x \neq 0$  since  $x \neq y \neq z$  given  
 $z-y \neq 0$

	$\therefore$ only $1 + xyz = 0$ $\Rightarrow xyz = -1$ Proved
Q.4)	Show $\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + a & b - ab \end{vmatrix} = (ab + bc + ca)^3$
Sol.4)	$R_1 \rightarrow aR_1; R_2 \rightarrow bR_2$ and $R_3 \rightarrow cR_3$ $= \frac{1}{abc} \begin{vmatrix} -abc & ab^2 + abc & ac^2 + abc \\ a^2b + abc & -abc & c^2b + abc \\ a^2c + abc & b^2c + abc & -abc \end{vmatrix}$ taking a, b, c common from $c_1, c_2$ and $c_3$ $= \frac{abc}{abc} \begin{vmatrix} -bc & ab + ac & ac + ab \\ ab + bc & -ac & bc + ab \\ ac + bc & bc + ac & -ab \end{vmatrix}$ $R_1 \rightarrow R_1 + R_2 + R_3$ $= \begin{vmatrix} ab + bc + ac & ab + bc + ac & ab + bc + ac \\ ab + bc & -ac & bc + ab \\ ac + bc & bc + ac & -ab \end{vmatrix}$ taking $(ab + bc + ca)$ common from $R_1$ $= (ab + bc + ca) \begin{vmatrix} 1 & 1 & 1 \\ ab + bc & -ac & bc + ab \\ ac + bc & bc + ac & -ab \end{vmatrix}$ $c_2 \rightarrow c_2 - c_1$ and $c_3 \rightarrow c_3 - c_1$ $= (ab + bc + ca) \begin{vmatrix} 1 & 0 & 0 \\ ab + bc & -ab - bc - ac & 0 \\ ac + bc & 0 & -ab - bc - ca \end{vmatrix}$ taking $(ab + bc + ca)$ common from $c_2$ and $c_3$ both $= (ab + bc + ca)(ab + bc + ca)^2 \begin{vmatrix} 1 & 0 & 0 \\ ab + bc - 10 & 0 & 0 \\ ac + bc & 0 & -1 \end{vmatrix}$ expanding along $R_1$ $= (ab + bc + ca)^3 (1) = (ab + bc + ca)^3 = \text{RHS}$
Q.5)	Show $\begin{vmatrix} b^2 + c^2 & ab & ac \\ ba & c^2 + b^2 & bc \\ ca & cb & a^2 + b^2 \end{vmatrix} = 4a^2b^2c^2$
Sol.5)	We have $\begin{vmatrix} b^2 + c^2 & ab & ac \\ ba & c^2 + b^2 & bc \\ ca & cb & a^2 + b^2 \end{vmatrix}$ $R_1 \rightarrow aR_1; R_2 \rightarrow bR_2$ and $R_3 \rightarrow cR_3$ $= \frac{1}{abc} \begin{vmatrix} a(b^2 + c^2) & a^2b & a^2c \\ ab^2 & b(c^2 + b^2) & b^2c \\ c^2a & c^2b & c(a^2 + b^2) \end{vmatrix}$ taking a, b, c common from $c_1, c_2, c_3$ resp. $= \frac{abc}{abc} \begin{vmatrix} b^2 + c^2 & a^2 & a^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$ $R_1 \rightarrow R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2(b^2 + c^2) & 2(c^2 + a^2) & 2(a^2 + b^2) \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$$

2 common from  $R_1$

$$= \begin{vmatrix} b^2 + c^2 & c^2 + a^2 & a^2 + b^2 \\ b^2 & c^2 + a^2 & b^2 \\ c^2 & c^2 & a^2 + b^2 \end{vmatrix}$$

$R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$

$$= 2 \begin{vmatrix} b^2 + c^2 & c^2 + a^2 & a^2 + b^2 \\ -c^2 & 0 & -a^2 \\ -b^2 & -a^2 & 0 \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_2 + R_3$

$$= 2 \begin{vmatrix} 0 & c^2 & b^2 \\ -c^2 & 0 & -b^2 \\ -b^2 & -a^2 & 0 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 0 & c^2 & b^2 \\ -c^2 & 0 & -a^2 \\ -b^2 & -a^2 & 0 \end{vmatrix}$$

expanding

$$= 2[-c^2(-a^2b^2) + b^2(a^2c^2)] \\ = 2(a^2b^2c^2 + a^2b^2c^2) = 4a^2b^2c^2 \quad \text{ans.}$$

Q.6)

Show that  $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

Sol.6)

We have  $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$

$c_1 \rightarrow c_1 + c_2 + c_3$

$$= \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & r+p & p+q \\ 2(x+y+z) & z+x & x+y \end{vmatrix}$$

$$= 2 \begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}$$

$c_2 \rightarrow c_2 - c_1$  and  $c_3 \rightarrow c_3 - c_1$

$$= 2 \begin{vmatrix} a+b+c & -b & -c \\ p+q+r & -q & -r \\ x+y+z & -y & -z \end{vmatrix}$$

Now,  $c_1 \rightarrow c_1 + c_2 + c_3$

$$= 2 \begin{vmatrix} a & -b & -c \\ p & -q & -r \\ x & -y & -z \end{vmatrix}$$

taking (-) sign from  $c_1$  &  $c_3$  both

$$= 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = \text{RHS}$$

Q.7)	Show $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$
Sol.7)	We have $\begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$ $R_1 \rightarrow R - 1 - xR_2$ $= \begin{vmatrix} a-ax^2 & c-cx^2 & p-px^2 \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$ $= \begin{vmatrix} a(1-x^2) & c(1-x^2) & p(1-x^2) \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$ taking $(1-x^2)$ common from $R_1$ $= (1-x^2) \begin{vmatrix} a & c & p \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix}$ $= R_2 \rightarrow R_2 - xR - 1$ $= (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix} = \text{RHS}$
Q.8)	Show that the value of the determinants $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.
Sol.8)	let $\Delta \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ $C_1 \rightarrow C_1 + C_2 + C_3$ $= \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a^2+b+c & a & b \end{vmatrix}$ (a + b + c) common from $C_1$ $= (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$ $R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$ $= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$ expanding along $R_1$ $= (a+b+c)(-a^2 - b^2 - c^2 + ab + bc + ca)$ $= -(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$ multiply & divide by 2 $= -\frac{1}{2}(a+b+c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$ $= -\frac{1}{2}(a+b+c)((a-b)^2 + (b-c)^2 + (c-a)^2)$ clearly the value of determinant is -ve ans.

Q.9)	<p>If <math>a, b, c</math> are real number such that <math>\begin{vmatrix} b+c &amp; c+a &amp; a+b \\ c+a &amp; a+b &amp; b+c \\ a+b &amp; b+c &amp; c+a \end{vmatrix} = 0</math> then show that either <math>a+b+c=0</math> (or) <math>a=b=c</math>.</p>
Sol.9)	<p>We have <math>\begin{vmatrix} b+c &amp; c+a &amp; a+b \\ c+a &amp; a+b &amp; b+c \\ a+b &amp; b+c &amp; c+a \end{vmatrix} = 0</math></p> <p><math>C_1 \rightarrow C_1 + C_2 + C_3</math></p> $\Rightarrow \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \\ 2(a+b+c) & b+c & c+a \end{vmatrix} = 0$ $\Rightarrow 2(a+b+c) \begin{vmatrix} 1 & c+a & a+b \\ 1 & a+b & b+c \\ 1 & b+c & c+a \end{vmatrix} = 0$ <p><math>R_2 \rightarrow</math> and <math>R_3 \rightarrow R_3 - R_1</math></p> $\Rightarrow 2(a+b+c) \begin{vmatrix} 1 & c+a & a+b \\ 0 & b-c & c-a \\ 0 & b-a & c-b \end{vmatrix} = 0$ <p>expanding along <math>R_1</math></p> $\Rightarrow 2(a+b+c)(-a^2 - b^2 - c^2 + ab + bc + ca) = 0$ $\Rightarrow -2(a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$ <p>multiply and divide by 2</p> $\Rightarrow -\frac{2}{2}(a+b+c)(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca) = 0$ $\Rightarrow -(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ $\Rightarrow (a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ $\Rightarrow \text{either } a+b+c = 0$ <p>(or) <math>(a-b)^2 + (b-c)^2 + (c-a)^2 = 0</math></p> <p>this is possible only when</p> $a-b = 0 \Rightarrow a = b$ $b-c = 0 \Rightarrow b = c$ $c-a = 0 \Rightarrow c = a$ $\Rightarrow a = b = c$ <p><math>\therefore</math> either <math>a+b+c = 0</math> (or) <math>a = b = c</math> ans.</p>
Q.10)	<p>Show that <math>\begin{vmatrix} 0 &amp; a &amp; -b \\ -a &amp; 0 &amp; -c \\ b &amp; c &amp; 0 \end{vmatrix} = 0</math></p>
Sol.10)	<p>let <math>\Delta = \begin{vmatrix} 0 &amp; a &amp; -b \\ -a &amp; 0 &amp; -c \\ b &amp; c &amp; 0 \end{vmatrix}</math></p> <p><math>R_1 \rightarrow cR_1; R_2 \rightarrow bcR_2</math> and <math>R_3 \rightarrow aR_3</math></p> $= \frac{1}{abc} \begin{vmatrix} 0 & ac & -bc \\ -a & b0 & -bc \\ ab & ac & 0 \end{vmatrix}$ <p>taking <math>ab, ac, bc</math> common from <math>C_1, C_2</math> &amp; <math>C_3</math></p> $= \frac{(ab)(ac)(bc)}{abc} \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix}$

$$= abc \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

expanding

$$= abc(0) = 0 = \text{RHS} \quad \text{ans.}$$

**Determinants****Class 12<sup>th</sup>****Solving System of Linear Equations (Matrix Method)**

Q.1)	An amount of Rs 5000 is put in to three investments at the rate of interest of 6%,7% and 8 % per annum. The total annual income is Rs 358. If the combined income from the first two investments is Rs 70 more than the income from the third, find the amount of each investment by matrix method.
Sol.1)	<p>Let Rs. <math>x</math> , Rs. <math>y</math> and Rs. <math>z</math> be the investments  from given conditions: <math>x + y + z = 5000</math> .....(1)  <math>\frac{6}{100} \times x + \frac{7}{100} \times y + \frac{8}{100} \times z = 358</math>  (or) <math>6x + 7y + 8z = 35800</math> .....(2)  and <math>\frac{6x}{100} + \frac{7y}{100} = \frac{8z}{100} + 70</math> {combined income from first two is 70 more than 3<sup>rd</sup>}  (or) <math>6x + 7y - 8z = 7000</math> .....(3)  <math>\therefore</math> the equations are  <math>x + y + z = 5000</math>  <math>6x + 7y + 8z = 35800</math>  <math>6x + 7y - 8z = 7000</math>  <math>\rightarrow</math> Now solve by yourself using <math>x = A^{-1}B</math>  <math>x = \text{Rs } 1000</math> ; <math>y = \text{Rs } 2200</math> ; <math>z = \text{Rs } 1800</math> ans.</p>
Q.2)	Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate Rs $x$ , Rs $y$ , Rs $z$ respectively per person. The first institution decided to award respectively 4 , 3 and 2 employees with a total prize money of Rs.37000 and the second institution decided to award respectively 5 , 3 and 4 employees with a total prize money of Rs.47000. If all the three prizes per person together amount to Rs.12000 then by matrix method. Find the value of $x$ , $y$ and $z$ .
Sol.2)	<p>Here Rs. <math>x</math> , Rs. <math>y</math> and Rs. <math>z</math> are the award money for resourcefulness , competence and determination respectively  from above data/condition , the equation are  <math>4x + 3y + 2z = 37000</math>  <math>5x + 3y + 4z = 47000</math>  <math>x + y + z = 12000</math>  (Do yourself using <math>X = A^{-1}B</math>)  Rs. 4000 , Rs. 5000 , Rs. 3000 ans.</p>
Q.3)	Two school's P and Q decided to award prizes for (1) academic (2) sports (3) all-rounder achievements. School P awarded Rs 12000 to 3, 1, 1 students while Q awarded Rs 7,000 to 1, 0, 2 students in the above categories. All the three prizes amount to Rs 6000. Find matrix representation of the above situation form equations and solve them by matrix method to find value of each prize. Do you agree that prizes should be given for honestly and good character also? Give reasons.
Sol.3)	<p>Let Rs. <math>x</math> , Rs. <math>y</math> , and Rs <math>z</math> are the awarded money for academic, sports and all rounder achievement respectively  the matrix form is  <math display="block">\begin{bmatrix} 3 &amp; 1 &amp; 1 \\ 1 &amp; 0 &amp; 2 \\ 1 &amp; 1 &amp; 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12000 \\ 7000 \\ 6000 \end{bmatrix}</math> (or) <math>A X = B \Rightarrow X = A^{-1}B</math>  Equations are <math>3x + y + z = 12000</math>  <math>x + 0y + 2z = 7000</math></p>



and  $x + y + z = 600$   
*Rs. 3000 , Rs 1000 and Rs 2000*      ans.

Properties of Determinates & Adjoint

Q.4) Show that  $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$  satisfies the equation  $f(x) = x^2 - 6x + 17 = 0$ . Hence find  $A^{-1}$ .

Sol.4) We have  $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$   
 $A^2 = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -5 & -18 \\ 18 & 7 \end{bmatrix}$   
 given  $f(x) = x^2 - 6x + 17$   
 $\Rightarrow f(A) = A^2 - 6A + 17I$   
 $= \begin{bmatrix} -5 & -18 \\ 18 & 7 \end{bmatrix} - \begin{bmatrix} 12 & -18 \\ 18 & 24 \end{bmatrix} + \begin{bmatrix} 17 & 0 \\ 0 & 17 \end{bmatrix}$   
 $A^2 - 6A + 17I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

Clearly A satisfies the equation  $x^2 - 6x + 17 = 0$

Now we have,  $A^2 - 6A + 17I = 0$

Pre-multiply by  $A^{-1}$

$$\Rightarrow A^{-1}A^2 - 6A^{-1}A + 17A^{-1}I = A^{-1}0$$

$$\Rightarrow A^{-1}A \cdot A - 6I + 17A^{-1} = 0$$

$$\Rightarrow IA - 6I + 17A^{-1} = 0$$

$$\Rightarrow A - 6I + 17A^{-1} = 0$$

$$\Rightarrow 17A^{-1} = 6I - A$$

$$\Rightarrow 17A^{-1} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{17} \begin{bmatrix} 4 & 3 \\ -3 & 2 \end{bmatrix} \quad \text{ans.}$$

Q.5) If  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$  show that  $A^3 - 6A^2 + 5A + 11I = 0$  and hence find  $A^{-1}$ .

Sol.5)  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$   
 $A^2 = A \cdot A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$   
 $A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$

Now  $A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6 \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 \quad (\text{proved})$$

(ii) we have  $A^3 - 6A^2 + 5A + 11I = 0$

Pre-multiply by  $A^{-1}$

$$\Rightarrow A^{-1}A^3 - 6A^{-1}A^2 + 5AA^{-1} + 11A^{-1}I = A^{-1}0$$

$$\Rightarrow A^{-1}A \cdot A^2 - 6A^{-1}A \cdot A + 5I + 11A^{-1}I = 0$$

$$\Rightarrow IA^2 - 6IA + 5I + 11A^{-1} = 0$$

$$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = 0$$

$$\Rightarrow 11A^{-1} = 6A - A^2 - 5I$$

$$= 6 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow 11A^{-1} = \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{11} \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix} \quad \text{ans.}$$

Q.6) If  $B \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$ . Find matrix B using inverse concept.

Sol.6) Let  $A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$  and  $C = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

then we have,  $ABA = C$

post multiply by  $A^{-1}$

$$\Rightarrow BAA^{-1} = CA^{-1}$$

$$\Rightarrow B I = CA^{-1}$$

$$\Rightarrow B = CA^{-1}$$

$$|A| = 4 + 2 = 6$$

$$(AdjA) = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6} \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Now  $B = CA^{-1}$

$$B = \frac{1}{6} \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \quad \text{ans.}$$

Q.7) Find matrix A if  $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ .

Sol.7) Let  $B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ ,  $C = \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix}$  and  $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

then we have  $BAC = D$

pre-multiply by  $B^{-1}$  and post multiply by  $C^{-1}$

$$\Rightarrow B^{-1} B A C C^{-1} = B^{-1} D C^{-1}$$

$$\Rightarrow I A I = B^{-1} D C^{-1}$$

$$\Rightarrow A = B^{-1} D C^{-1}$$

$$B^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$

and  $C^{-1} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$  } (find yourself)

$$A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \quad \text{Ans...}$$

Q.8) If  $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$  verify that  $(AB)^{-1} = B^{-1} A^{-1}$ .

Sol.8)  $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$  &  $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

$$|A| = 15 - 14 = 1$$

$$|B| = 54 - 56 = -2$$

$$Adj A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$Adj B = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B^{-1} = -\frac{1}{2} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

$$\text{Now } AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} 34 & 39 \\ 82 & 94 \end{bmatrix}$$

$$|AB| = 34 \times 94 - 82 \times 39 = -2$$

$$\text{Adj}(AB) = \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$$

$$(AB)^{-1} = -\frac{1}{2} \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$$

Taking RHS =  $B^{-1}A^{-1}$

$$= -\frac{1}{2} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix} = (AB)^{-1}$$

Hence  $(AB)^{-1} = B^{-1}A^{-1}$  verified ans.

Q.9) If  $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$  show that  $A' A^{-1} = \begin{bmatrix} \cos(2x) & -\sin(2x) \\ \sin(2x) & \cos(2x) \end{bmatrix}$

Sol.  $A' = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$

$$|A| = 1 + \tan^2 x$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A = \frac{1}{1 + \tan^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

Taking LHS  $A' A^{-1}$

$$= \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \frac{1}{1 + \tan^2 x} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$= \frac{1}{1 + \tan^2 x} \begin{bmatrix} 1 - \tan^2 x & -2 \tan x \\ 2 \tan x & 1 - \tan^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1 - \tan^2 x}{1 + \tan^2 x} & \frac{-2 \tan x}{1 + \tan^2 x} \\ \frac{2 \tan x}{1 + \tan^2 x} & \frac{1 - \tan^2 x}{1 + \tan^2 x} \end{bmatrix}$$

$$= \begin{bmatrix} \cos(2x) & -\sin(2x) \\ \sin(2x) & \cos(2x) \end{bmatrix} = \text{RHS ans.}$$

- Q.10) (a) Find area of  $\Delta ABC$  whose vertices are  $A(3, 8)$ ,  $B(-4, 2)$ ,  $C(5, -1)$ .  
 (b) Find equation of line joining  $A(3, 5)$  &  $B(4, 2)$  using determinants.  
 (c) Find value of  $\lambda$  so that points  $(1, -5)$ ,  $(-4, 7)$  and  $(\lambda, 7)$  are collinear.

Sol.10) (a)  $A(3, 8)$ ,  $B(-4, 2)$ ,  $C(5, -1)$

$$\text{Area of } \Delta ABC = \frac{1}{2} \begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

$$= \frac{1}{2} |3(2 + 1) - 8(-4 - 5) + 1(4 - 10)|$$

$$= \frac{1}{2} |9 + 72 - 6| = \frac{75}{2} \text{ square units}$$

(b) equation of  $AB$  is given by

$$\begin{vmatrix} x & y & 1 \\ 3 & 5 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow x(5 - 2) - y(3 - 4) + 1(6 - 20) = 0$$

$$\Rightarrow 3x + y - 14 = 0 \quad \text{ans.}$$

(c) since  $(1, -5)$ ,  $(-4, 5)$  and  $(\lambda, 7)$  are collinear

Area of  $\Delta = 0$

$$\therefore \frac{1}{2} \begin{vmatrix} 1 & -5 & 1 \\ -4 & 5 & 1 \\ \lambda & 7 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 1(5 - 7) + 5(-4 - \lambda) + 1(-28 - 5\lambda) = 0$$

$$\Rightarrow -2 - 20 - 5\lambda - 28 - 5\lambda = 0$$

$$\Rightarrow -10\lambda = -50$$

$$\lambda = -5 \quad \text{ans.}$$

## Determinants Class 12<sup>th</sup>

Q.1)	Find the value of x if the area of $\Delta$ is 35 square units with vertices $(x, 4)$ , $(2, -6)$ and $(5, 4)$ .
Sol.1)	<p>Let vertices are <math>A(x, 4)</math>, <math>B(2, -6)</math> and <math>C(5, 4)</math></p> <p>Area of <math>\Delta ABC = \frac{1}{2} \begin{vmatrix} x &amp; 4 &amp; 1 \\ 2 &amp; -6 &amp; 1 \\ 5 &amp; 4 &amp; 1 \end{vmatrix}</math></p> <p><math>35 = \frac{1}{2}  x(-10) - 4(-3) + 1(38) </math></p> <p><math>\Rightarrow 35 = \frac{1}{2}  -10x + 12 + 38 </math></p> <p><math>\Rightarrow 70 =  -10x + 50 </math></p> <p><math>70 = -10x + 50 \quad \left  \quad -70 = -10x + 50 \right.</math></p> <p><math>10x = -20 \quad \left  \quad 10x = 120 \right.</math></p> <p><math>x = -2 \quad \left  \quad x = 12 \right.</math></p> <p><math>\therefore x = -2, x = 12</math> ans.</p>
Q.3)	<p>Find the value of x so that matrix <math>A = \begin{bmatrix} (x-1) &amp; 1 &amp; 1 \\ 1 &amp; (x-1) &amp; 1 \\ 1 &amp; 1 &amp; (x-1) \end{bmatrix}</math> is singular/ Non-Invertible.</p>
Sol.3)	<p>Since matrix A is singular</p> <p><math>\therefore  A  = 0</math></p> <p><math>\begin{vmatrix} x-1 &amp; 1 &amp; 1 \\ 1 &amp; x-1 &amp; 1 \\ 1 &amp; 1 &amp; x-1 \end{vmatrix} = 0</math></p> <p><math>\Rightarrow (x-1)[(x-1)^2 - 1] - 1[x-1-1] + 1[1-x+1] = 0</math></p> <p><math>\Rightarrow (x-1)(x^2 - 2x) - 1(x-2) + (2-x) = 0</math></p> <p><math>\Rightarrow x^3 - 2x^2 - x^2 + 2x - x + 2 + 2 - x = 0</math></p> <p><math>\Rightarrow x^3 - 3x^2 + 4 = 0</math></p> <p>By trial method</p> <p><math>(x+1)(x-2)(x+1) = 0</math></p> <p><math>\Rightarrow x = -1, x = 2</math> ans.</p>
Q.4)	<p>(a) Evaluate the determinant <math>\Delta = \begin{vmatrix} 1 &amp; \sin \theta &amp; 1 \\ -\sin \theta &amp; 1 &amp; \sin \theta \\ -1 &amp; -\sin \theta &amp; 1 \end{vmatrix}</math>. Also prove <math>2 \leq \Delta \leq 4</math>.</p> <p>(b) Prove that <math>\Delta = \begin{vmatrix} x &amp; \sin \theta &amp; \cos \theta \\ -\sin \theta &amp; -x &amp; 1 \\ \cos \theta &amp; 1 &amp; x \end{vmatrix}</math> is independent of <math>\theta</math>.</p>
Sol.4)	<p>(a) we have, <math>\Delta = \begin{vmatrix} 1 &amp; \sin \theta &amp; 1 \\ -\sin \theta &amp; 1 &amp; \sin \theta \\ -1 &amp; -\sin \theta &amp; 1 \end{vmatrix}</math></p> <p><math>\Rightarrow \Delta = 1(1 + \sin^2 \theta) - \sin \theta(-\sin \theta + \sin \theta) + 1(\sin^2 \theta + 1)</math></p> <p><math>\Rightarrow \Delta = 1 + \sin^2 \theta + 0 + \sin^2 \theta + 1</math></p> <p><math>\Rightarrow \Delta = 2 + 2 \sin^2 \theta</math></p> <p>Now, we know</p> <p><math>-1 \leq \sin \theta \leq 1</math></p> <p><math>\Rightarrow 0 \leq \sin^2 \theta \leq 1</math></p> <p><math>\Rightarrow 0 \leq 2 \sin^2 \theta \leq 2</math> (multiply by 2)</p> <p><math>\Rightarrow 2 \leq 2 + 2 \sin^2 \theta \leq 4</math> (adding 2)</p> <p><math>\Rightarrow 2 \leq \Delta \leq 4</math> (proved)</p> <p>(b) <math>\Delta = x(-x^2 - 1) - \sin \theta(-x \sin \theta - \cos \theta) + \cos \theta(-\sin \theta + x \cos \theta)</math></p> <p><math>\Delta = -x^3 - x + x \sin^2 \theta + \sin \theta \cos \theta - \sin \theta \cos \theta + x \cos^2 \theta</math></p>

$\Delta = -x^3 - x + x(\sin^2\theta + \cos^2\theta)$ $\Delta = -x^3 - x + x(1)$ $\Delta = -x^3 \text{ which is independent of } \theta.$
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## Short Questions

- Q.5) Order  $3 \times 3$ ,  $|A| = 5$ . Find  $|\text{Adj } A| = ?$   
 Sol.5) We have  $n = 3$ ,  $|A| = 5$   
 and  $|\text{Adj } A| = |A|^{n-1}$   
 $= (5)^{3-1} = 25$  ans.
- Q.6) Order  $3 \times 3$ ,  $|\text{Adj } A| = 81$  find  $|A| = ?$   
 Sol.6) We have  $n = 3$ ,  $|\text{Adj } A| = 81$   
 $\Rightarrow |\text{Adj } A| = |A|^{n-1}$   
 $\Rightarrow 81 = |A|^2$   
 $\Rightarrow |A| = \pm 9$  ans.
- Q.7) Order  $3 \times 3$ ;  $|A| = 3$  find  $|4A| = ?$   
 Sol.7) We have  $n = 3$ ,  $|A| = 3$   
 $|4A| = 4^3 |A| \quad \dots\{\because |kA| = k^n |A|\}$   
 $= 64 \times 3$   
 $= 192$  ans.
- Q.8) Order  $3 \times 3$ ;  $|A| = 5$  find  $|2\text{Adj } A| = ?$   
 Sol.8)  $|2\text{Adj } A| = 2^3 |\text{Adj } A| = 2^3 |A|^{3-1}$   
 $= 8 (5)^2 = 200$  ans.
- Q.9) Order  $4 \times 4$ ;  $|3 \text{Adj } A| = 243$  Find  $|A| = ?$   
 Sol.9) We have  $|3 \text{Adj } A| = 3^4 |\text{Adj } A|$   
 $243 = 3^4 |A|^{4-1}$   
 $243 = 81 |A|^3$   
 $|A|^3 = 3$   
 $|A| = (3)^{1/3}$  ans.
- Q.10) Order  $4 \times 4$ ;  $|A| = 5$  find  $|A'| = ?$   
 Sol.10) We know  $|A'| = |A|$   
 $\Rightarrow |A'| = 5$  ans.

**CLASS XII**  
**Continuity and Differentiability**

One marks questions

Differentiate the following with respect to x:

1.  $2^{\cos^2 x}$                       Ans:-  $2^{\cos^2 x} \sin 2x \log 2$  (NCERT EXEMPLAR)
2.  $\log(x + \sqrt{a + x^2})$                       Ans:  $\frac{1}{\sqrt{x^2+a}}$  (NCERT EXEMPLAR)
3.  $\sqrt{e^{\sqrt{x}}}$                       Ans:  $\frac{e^{\sqrt{x}}}{4\sqrt{x}e^{\sqrt{x}}}$

Four marks questions

1. If  $\sin y = x \sin(a + y)$ , prove that  $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$
2. If  $y = x \sin y$ , prove that  $\frac{dy}{dx} = \frac{y}{x(1 - x \cos y)}$ .
3. If  $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$ , prove that  $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$
4. If  $x = a(\cos t + t \sin t)$  and  $y = a(\sin t - t \cos t)$ , find the value of  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ , when  $t = \frac{\pi}{4}$ .
5. If  $y = \log[x + \sqrt{x^2 + 1}]$ , prove that  $(x^2 + 1)y_2 + xy_1 = 0$ .
6. If  $y = e^{a \cos^{-1} x}$ ,  $-1 \leq x \leq 1$ , show that  $(1 - x^2)y_2 - xy_1 - a^2 y = 0$ .
7. Differentiate with respect to x:  
a)  $\tan^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$                       Ans:  $\frac{dy}{dx} = -\frac{1}{2}$
8. If  $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ , prove that  $\frac{dy}{dx} = \frac{4}{1+x^2}$ .
9. Verify Rolle's theorem for the function  $f(x) = \sin x - \sin 2x$  on  $[0, \pi]$ .
10. Verify Mean Value theorem for the function  $f(x) = x^2 - 2x + 4$  on  $[1, 5]$ .
11. Find the values of 'a' and 'b' when  

$$f(x) = \begin{cases} 3ax + b, & x > 1 \\ 11, & x = 1 \\ 5ax - 2b, & x < 1 \end{cases}$$
is continuous at  $x = 1$ . Ans:  $a = 3, b = 2$ .
12. If  $x^{13} \cdot y^7 = (x + y)^{20}$ , prove that  $\frac{dy}{dx} = \frac{y}{x}$ .
13. If  $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \text{to } \infty}}}$ , prove that  $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$ .

14. Check, the function  $f(x) = \begin{cases} |x - a| \sin \frac{1}{x - a}, & \text{if } x \neq 0 \\ 0, & \text{if } x = a \end{cases}$  is continuous or discontinuous at  $x = a$ . (NCERT EXEMPLAR).

15. Examine the differentiability of  $f$ , where  $f$  is defined by

$$f(x) = \begin{cases} x[x], & \text{if } 0 \leq x < 2 \\ (x - 1)x, & \text{if } 2 \leq x < 3 \end{cases} \quad \text{at } x = 2, \text{ (NCERT EXEMPLAR).}$$

16. Find  $\frac{dy}{dx}$  of the function expressed in parametric form give  $x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2 \log t}{t}$ . (NCERT EXEMPLAR). Ans :  $t$

17. Find  $\frac{dy}{dx}$  when  $x$  and  $y$  are connected by the relation given by  $\sin(xy) + \frac{x}{y} = x^2 - y$  (NCERT EXEMPLAR). Ans:  $\frac{2xy^2 - y^3 \cos(xy) - y}{xy^2 \cos(xy) - x + y^2}$

18. Discuss the applicability of Rolle's Theorem on the function given

$$\text{by } f(x) = \begin{cases} x^2 + 1, & \text{if } 0 \leq x \leq 1 \\ 3 - x, & \text{if } 1 \leq x \leq 2 \end{cases} \quad \text{(NCERT EXEMPLAR)}$$

**CLASS – XII SUBJECT – MATHEMATICS ASSIGNMENT NO.3 TOPIC – CONTINUITY & DIFFERENTIATE**

- Find the point of discontinuity for the function  $f(x) = \begin{cases} x^4 - 16 & x \neq 0 \\ x - 2 & x = 0 \end{cases}$
- Show that  $f(x) = \begin{cases} 5x - 4 & a < x < 1 \\ 4x^3 - 3x & 1 < x < 2 \end{cases}$  is continuous at  $x = 1$
- For what value of K in the for continuous  $x = 0$ ,  $f(x) = \frac{1 - \cos 4x}{8x^2}$   $x \neq 0$
- If  $f(x) = \begin{cases} 3ax + b & \text{if } x > 1 \\ 11 & \text{if } x = 1 \end{cases}$   $5ax - 2b$  if  $x < 1$  is continuous at  $x = 1$ . find a, b
- If  $f(x)$  is differentiable at  $x = a$  find  $\lim_{x \rightarrow a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$
- Find values of  $a, b$  that  $f(x)$  given by  $f(x) = \begin{cases} 1 & : f(x) \leq 3 \\ ax + b & 3 < x < 5 \\ 7 & \text{if } x \leq 5 \end{cases}$
- If  $y = -\cot^2 \frac{x}{2} - x$  by  $\sin \frac{x}{2}$ , prove :  $\frac{dy}{dx} = \cot 3 \frac{x}{2}$
- If  $x = \cos \theta$  and  $y = \sin \theta - \theta \cos \theta$ , prove  $\frac{d^2 y}{dx^2} = \frac{\sec^2 \theta}{\theta}$
- If  $y = \frac{2 - 3 \cos x}{\sin x}$ , find  $\frac{dy}{dx}$  at  $x = \frac{\pi}{6}$
- If  $y = 10g(1 + \cos x)$ , Prove  $\frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} \cdot \frac{dy}{dx} = 0$
- If  $y = \tan^{-1} \frac{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}{\sqrt{a^2 + x^2} - \sqrt{a^2 - x^2}}$  show that  $\frac{dy}{dx} = \frac{-2a^2}{x^3} \left[ 1 + \frac{a^2}{\sqrt{a^4 - x^4}} \right]$
- If  $Y = \sqrt{\frac{1 + e^x}{1 - e^x}}$ , then show that  $\frac{dy}{dx} = \frac{e^x}{(1 - e^x)\sqrt{1 - e^{2x}}}$
- If  $y = \sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$  find  $\frac{dy}{dx}$
- If  $y = \sin^{-1} \left( \frac{x^2 - y^2}{x^2 + y^2} \right) = \tan^{-1} a$ , prove :  $\frac{dy}{dx} = \frac{y}{x}$
- If  $e^y = y^x$ , prove  $\frac{dy}{dx} = \frac{(\log y)^2}{\log y - 1}$
- If  $x = a(\cos t + \log \tan t/2)$ ,  $y = \sin t$  then find  $\frac{dy}{dx}$  at  $t = \frac{\pi}{4}$
- Differentiate  $e^{\sin x} + (ax)^x$  w.r.t.  $x$ .
- If  $y = e^x (\sin x + \cos x)$  Prove :  $\frac{d^2 y}{dx^2} - \frac{2dy}{dx} + 2y = 0$
- If  $x = 3 \sin t - \sin 3t$ ,  $y = 3 \cos t - \cos 3t$  find  $\frac{d^2 y}{dx^2}$  at  $t = \frac{\pi}{4}$



20. Find  $dy$  if (i)  $x = a + t^2$ ,  $y = 2t$  (ii)  $x = 1 + 10gt$ ,  $y = 2 \sin \square$
21. Find  $\frac{dy}{dx}$ , if (i)  $x = a \frac{(1+t^2)}{1-t^2}$ ,  $y = \frac{2t}{1-t^2}$ , (ii)  $x = \frac{1 + \log t}{E^2}$ ,  $y = 2 \sin \square - \sin^2 \square$  (iii)  $x^y = y^x$
22. If  $e^y (x+1) = 1$ , Prove that  $\frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2$
23. If  $x^4 = c^{x-y}$ , then prove that  $\frac{dy}{dx} = \frac{\log x}{\log(ex)^2}$
24. If  $\sin(x, y) = x^2 - y$ , find  $\frac{dy}{dx}$
25. If  $e^x + e^y = e^{x+y}$  prove that  $\frac{dy}{dx} = e^{y-x}$
26. Find  $\frac{dy}{dx}$  if  $\cos(x+y) = y \sin a$ .
27. If  $x \sqrt{1+y} = y \cdot \sqrt{1+x} = 0$  then prove that  $\frac{dy}{dx} = \frac{-1}{\sqrt{1+x^2}}$
28. If  $x \sqrt{1-y^2} + y \sqrt{1-x^2} = 1$ , then prove  $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$
29. If  $y = x(a+y)$  show:  $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$
30. If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ , prove  $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

1. Examine the continuity of the function  $f(x) = \begin{cases} 1+x; x \leq 2 \\ 5-x; x > 2 \end{cases}$  at  $x=2$ .

2. Show that the function  $f(x) = \{2x - |x|\}$  is continuous at  $x=0$ .

3. Discuss the continuity of the function:  $f(x) = \begin{cases} \frac{e^x - 1}{\log(1+2x)}, x \neq 0 \\ 7, x = 0 \end{cases}$

at the point  $x=0$ .

4. Examine the continuity of the functions at  $x=0$

$$(i) f(x) = \begin{cases} \frac{x}{\sin 3x}, x \neq 0 \\ 3, x = 0 \end{cases} \quad (ii) f(x) = \begin{cases} \frac{\sin 2x}{\sin 3x}, x \neq 0 \\ 2, x = 0 \end{cases}$$

5. Determine the constants if the given functions are continuous at specified points

$$(i) f(x) = \begin{cases} ax+5; x \leq 2 \\ x-1, x > 2 \end{cases} \text{ at } x=2 \quad (ii) f(x) = \begin{cases} \frac{x^2 - 3x + 2}{x-1}, x \neq 1 \\ k, x = 1 \end{cases} \text{ at } x=1$$

6. Find the value of a,b and c for which the function is continuous at  $x=0$ .

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, x < 0 \\ c, x = 0 \\ \frac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}}, x > 0 \end{cases}$$

7. Find the derivative of the function with respect to x

$$(i) e^{\tan^{-1}(\cos \sqrt{x})} \quad (ii) \log\left(\frac{x^2+x+1}{x^2-x+1}\right) + \frac{2}{\sqrt{3}} \tan^{-1} \frac{\sqrt{3}x}{1-x^2}$$

8. If  $y = x \log\left(\frac{x}{a+bx}\right)$ , show that  $x^3 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^3$

9. If  $y = 3 \cos t - \cos 3t$ ,  $x = 3 \sin t - \sin 3t$ , find  $\frac{d^2y}{dx^2}$  at  $t = \frac{\pi}{3}$

10. If  $ax^2 + 2hxy + by^2 = 1$ , show that  $\frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx+by)^3}$

**Sant Nirankari Public School, Faridabad**  
**Class- XII**  
**Subject- Business Studies**  
**Chapter 1- Nature and Significance of Management**

**Notes**

**Date- 20<sup>th</sup> March,2020**

**Concept of Management**

Management is an art of getting things done with and through others. Management can be defined as, the process of getting things done with the aim of achieving organizational goals effectively and efficiently.

**EFFICIENCY AND EFFECTIVENESS**

Efficiency (completing the work at low cost) means doing the task correctly at minimum cost through optimum utilization of resources while effectiveness (Completing the work on time) is concerned with end result means completing the task correctly within stipulated time. Although efficiency and effectiveness are different yet they are inter related. It is important for management to maintain a balance between the two.

1. Rohini prepared a well-documented and factual report on Co's performance but she could not present it in Board meeting as she could not complete it on time.

[Hint: Efficient but not effective]

2. Best roadways promised to deliver goods in time and charged extra money from Mr. Singh. But the goods were not delivered on time.

[Hint: Efficient but not effective]

**CHARACTERISTICS OF MANAGEMENT**

1. Goal oriented Process

It is a goal oriented process, which is to achieve already specified and desired objectives by proper utilization of available resources.

2. Pervasive:

Management is universal in nature. It is used in all types of organizations whether economic, social or political irrespective of its size, nature and location and at each and every level.

3. Multidimensional:

It is multidimensional as it involves management of work, people and operations.

4. Continuous:

It consists of a series of function and its functions are being performed by all managers simultaneously. The process of management continues till an organization exists for attaining its objectives.

5. Group Activity:

It is a group activity since it involves managing and coordinating activities of different people as a team to attain the desired objectives of the organization.

6. Dynamic function :

It is a dynamic function since it has to adapt according to need, time and situation of the changing business environment. For example, McDonalds made major changes in its 'Menu' to survive in the Indian market.

7. Intangible Force:

It is intangible force as it can't be seen but its effects can be felt in the form of results like whether the objectives are met and whether people are motivated or not and there is orderliness and coordination in the work environment.

**Date – 21<sup>st</sup> March,2020**

**Assignment 1**

1. Alexa Textiles Ltd.'s target is to produce 20,000 shirts per month at a cost of 500 per shirt. The production manager achieved this target at a cost of 450 per shirt. Do you think the manager is effective or efficient or both ? Give reasons to support your answer.

2. Mr. A drafted a well documented and factual report on the financial performance of the company but he could not present it in the Board Meeting as he could not complete it till the day of meeting. Is he effective, efficient , both or neither ? Give reasons.

3. Mr. B of fast Couriers assured to deliver an urgent consignment of its regular customer on 7th April,2018 at an extra cost of 1,000. The consignment was delivered on 8th April,2018. Is Mr. B efficient, effective, both or neither ? Give reasons.

4. Identify the characteristic of management highlighted in the following statements :

- i) In an organization, the employees are happy and satisfied, there is no chaos and the effect of management is noticeable.
- ii) An educational institution as well as a business organization both need to be managed.
- iii) In order to be successful an organization must change its goals according to the needs of environment.
- iv) Pizza hut keeps on introducing new variety of pizzas in their menu.
- v) Management consists of ongoing series of functions.
- vi) Management sets targets and unites effort of all individuals to achieve them,
- vii) The task of management is to make people work towards achieving the organisation's goals by making their strengths effective and their weaknesses irrelevant.
- viii) Team work and coordination is needed to fulfil the common organizational goal.

5. Amit joined as a production manager of a reputed company. During the induction training, he was being told that the company wanted to achieve 30% increase in output in the next quarter. So, he was expected to make production plans, identify incentive schemes for workers to make their strengths effective and ensure there is no technical glitch. Amit realized quickly that his job is a series of ongoing tasks. After one month, he was informed by the General manager that due to increase in international demand, production targets have been raised. He also had a very good rapport with his team as he always emphasizes on unity and harmony among the team. So, he immediately called a meeting and asked his team to work overtime to meet the extended target. He was delighted that everyone agreed and at the end of quarter, he was able to meet the targets, workers were happy and there was no chaos.

Identify the features/characteristics of management highlighted by quoting the lines.

**Date- 23<sup>rd</sup> March,2020**

**Notes**

## **OBJECTIVES OF MANAGEMENT**

### **(1) Organizational objectives:**

Organizational Objectives can be divided into Survival (Earning enough revenues to cover cost); Profit (To cover cost and risk); and Growth (To improve its future prospects).

(A) Survival – Management by taking positive decisions with regard to different business activities ensures survival of business for long term.

(B) Profit – It plays an important role in facing business risks and successful running of business activities.

(C) Growth – Management must ensure growth which can be measured by increase in sales, number of employees, number of products, additional investment, etc.

### **(2) Social Objectives:**

Social objectives is to provide some benefits to society like applying environmental friendly practices in the production process and giving employment to disadvantaged sections of society, etc. Example: TISCO,ITC, and Asian Paints.

### **(3) Personal Objectives:**

Personal Objectives is to focus on diverse personal objectives of people working in the organization which need to be reconciled with organizational objectives.

## **Importance of Management**

(1) Achieving Group Goals: Management creates team work and coordination in the group. Managers give common direction to individual efforts in achieving the overall goals of the organization.

2) Increases Efficiency: Management increases efficiency by using resources in the best possible manner to reduce cost and increase productivity.

(3) Creates Dynamic organization: Management helps the employees overcome their resistance to change and adapt as per changing situation to ensure its survival and growth.

(4) Achieving personal objectives: Management helps the individuals achieve their personal goals while working towards organizational objectives.

(5) Development of Society: Management helps in the development of society by producing good quality products, creating employment opportunities and adopting new technologies.

**Date- 24<sup>th</sup> March, 2020**  
**Assignment 2**

1. Naya Subah Ltd. is an upcoming organization producing air purifiers. The company is earning sufficient profits and also able to save enough to invest in various projects. The management of the company believes that a satisfied workforce creates a satisfied customer, who in turn creates profits that lead to satisfied shareholders. So, it pays competitive salaries and perks to all its employees and also provided various opportunities for their personal development and growth. This company is also acting as a responsible member of society as it has set up an engineering college for students belonging to lower strata and gives 50% scholarship to deserving students.

Is the management of Naya Subah Ltd. fulfilling its objectives? Justify your answer by quoting the lines from the para.

2. Identify the objectives from the following statements :

- i) It is essential to meet the costs of business and also covers the business risks.
- ii) A company uses environment friendly method of production.
- iii) Management provided peer recognition.
- iv) Company uses eco friendly materials to manufacture the product.
- v) Management provides competitive salaries.
- vi) Company is able to increase its products and number of employees.
- vii) Management is ensuring to provide safe working conditions for the employees.
- viii) Organization has launched cleanliness drive for the nearby areas.
- ix) Company is earning sufficient enough to cover the costs.

3. Identify the importance of management from the following statements :

- i) The aim of manager is to reduce costs and increase productivity.
- ii) Management helps to provide good quality products and services, creates employment opportunities.
- iii) Management helps people to adapt to the changes so that the organization is able to maintain its competitive edge.
- iv) A manager motivates and lead his team in such a manner that individual members are able to achieve personal goals while contributing to the overall organizational objectives.
- v) The task of a manager is to give a common direction to the individual effort in achieving the overall goals of the organization.

4. Ahmed Qureshi works as a manager of a reputed company in power sector. He is an efficient manager and as a result of his excellent managerial competence, the company is able to reduce costs and increase productivity. The company belongs to infrastructure sector wherein regular amendments are made in the govt. regulations and policies. He holds regular meetings to ensure that people in his department are not only aware of the related changes but are also able to adopt these changes effectively. This helps the to maintain competitive edge. He motivates and leads his team in such a manner that individual members are able to achieve personal goals while contributing to the overall organizational objectives. In the process of fulfilling his duties for the growth of the organization, he helps in providing competitive services, creating new employment opportunities etc. for the greater good of the people at large.

Identify the points of importance of management by quoting the lines from para.

**Date- 25<sup>th</sup> March,2020**

**Notes**

### **Management as an Art**

Art refers to skillful and personal application of existing knowledge to achieve desired results. It can be acquired through study, observation and experience. The features of art as follows:

- (1) Existence of theoretical knowledge: In every art, Systematic and organized study material should be available compulsorily to acquire theoretical knowledge.
- (2) Personalized application: The use of basic knowledge differs from person to person and thus, art is a very personalized concept.
- 3) Based on practice and creativity: Art involves in consistent and creative practice of existing theoretical knowledge.

In management also a huge volume of literature and books are available on different aspects of management. Every manager has his own unique style of managing things and people. He uses his creativity in applying management techniques and his skills improve with regular application. Since all the features of art are present in management. so it can called an art.

### **Management as a Science**

Science is a systematized body of knowledge that is based on general truths which can be tested anywhere, anytime. The features of Science are as follows:

- 1) Systematized body of knowledge: Science has a systematized body of knowledge based on principles and experiments.
- (2) Principles based on experiments and observation: Scientific principles are developed through experiments and observation.
- (3) Universal validity: Scientific principles have universal validity and application.

Management has systematic body of knowledge and its principles are developed over a period of time based on repeated experiments & observations which are universally applicable but they have to be modified according to given situation.

As the principles of management are not as exact as the principles of pure science, so it may be called-an inexact science. The prominence of human factor in the management makes it a Social Science.



**Date-26<sup>th</sup> March, 2020**

**Notes**

### **Management as Profession**

Profession means an occupation for which specialized knowledge and skills are required and entry is restricted. The main features of profession are as follows:

(1) Well-defined body of Knowledge: All the professions are based on well defined body of knowledge.

(2) Restricted Entry: The entry in every profession is restricted through examination or through some minimum educational qualification.

(3) Professional Associations: All professions are affiliated to a professional association which regulates entry and frames code of conduct relating to the profession.

(4) Ethical Code of Conduct: All professions are bound by a code of conduct which guides the behaviour of its members.

(5) Service Motive: The main aim of a profession is to serve its clients.

Management does not fulfill all the features of a profession and thus it is not a full-fledged profession like doctor, lawyer, etc., but very soon it will be recognized as full-fledged profession.

**Date-27<sup>th</sup> March,2020**

**Notes**

**Levels of Management:**

Top, Middle and Operational Levels

“Levels of management” means different categories of managers, the lowest to the highest on the basis of their relative responsibilities, authority and status.

**Top Level**

Consists of Chairperson, Chief Executive Officer, Chief Operating Officer or equivalent and their team.

Chief task is to integrate and to coordinate the various activities of the business, framing policies, formulating organizational goals & strategies.

**Middle Level**

Consists of Divisional or Departmental heads, Plant Superintendents and Operation Managers etc.

Main tasks are to interpret the policies of the top management to ensure the availability of resources to implement policies, to coordinate all activities, ensure availability of necessary personnel & assign duties and responsibilities to them.

**Lower Level/Supervisory Level**

Consists of Foremen and supervisor etc. Main task is to ensure actual implementation of the policies as per directions, bring workers’ grievances before the management & maintain discipline among the workers.

**Date- 28<sup>th</sup> March,2020**

**Notes**

### **Functions of Management**

- 1.Planning: Thinking in advance what to do, when to do, and who is going to do it. It bridges the gap between where we are and where we want to reach.
- 2.Organising: organization means deciding the framework of working how many units and sub-units are needed,how many posts are needed, how to distribute the authority and responsibilities.
3. Staffing: It refers to recruitment, selection, training, development and appointment of the employees.
- 4.Directing: It refers to guiding, instructing, inspiring and motivating the employees.
- 5.Controlling are the main functions of management. Controlling is monitoring the organizational performance towards the attainment of the organizational goals.

**Date- 29<sup>th</sup> March,2020**

**Assignment 3**

1. At which level of management are the managers responsible for maintaining the quality of output and safety standards?
2. State three functions of middle level of management.
3. "Science is a systematised body of knowledge that explains certain general truths or the operations of general laws." In the light of this statement describe whether management is a science.
4. Write the functions of Top Level Management.
5. Discuss the basic features of management as a profession.
6. Management is a series of continuous inter-related functions. Comment.
7. \_\_\_\_\_ function of management examines the activities and resources required to implement the plan. ( fill in the blank)
8. 'Planning cannot prevent problems.' (True/ False)
9. Which function of management is concerned with finding right people for the right job?
10. Planning is the function of determining in advance what is to be done and who has to do it. ( True/ False).

**Date- 30<sup>th</sup> March,2020**

**Notes**

**Coordination (The Essence of Management):**

Coordination is the force which synchronizes all the functions of management and activities of different departments. Lack of coordination results in overlapping, duplication, delays and chaos. It is concerned with all the three levels of management as if all the levels of management are looked at together, they become a group and as in the case of every group, they also require coordination among themselves. So, it is not a separate function of management, rather it is the essence of management.

1. Coordination integrates group efforts: It integrates diverse business activities into purposeful group activity ensuring that all people work in one direction to achieve organizational goals.
2. Coordination ensures unity of action: It directs the activities of different departments and employees towards achievement of common goals and brings unity in individual efforts.
3. Coordination is a continuous process: It is not a specific activity matter it is required at all levels, in all departments till the organization continues its operations.
4. Coordination is all pervasive function: It is universal in nature. It synchronizes the activities of all levels and departments as they are interdependent to maintain organizational balance.
5. Coordination is the responsibility of all managers: It is equally important at all the three-top, middle and lower levels of management. Thus it is the responsibility of all managers that they make efforts to establish coordination.
6. Coordination is a deliberate function: Coordination is never established by itself rather it is a conscious effort on the part of every manager. Cooperation is voluntary effort of employees to help one another. Effective coordination cannot be achieved without cooperation of group members.

**Date- 31<sup>st</sup> March,2020**

**Assignment 4**

Q1. Management defined as a process of getting things done with the aim of achieving goals effectively and efficiently .Process in the definition means the primary function or activities that management performs to get things done. Enumerate the functions or activities.

Q.2 The basic objective of any business is\_\_\_\_\_

- |              |              |
|--------------|--------------|
| (a) Survival | (c) Growth   |
| (b) Profit   | (d) Personal |

Q3. \_\_\_\_\_ is responsible for implementing and controlling plans and strategies developed by the senior most executives of the organisation.(Fill up the blank with correct answer)

Q4 The purchase, production and sales managers at Sharda Ltd, a firm manufacturing ready made garments are generally at a conflict, as they have their own objectives. Usually each thinks that only they are qualified to evaluate, judge and decide on any manner, according to their professional criteria.

Name the concept which will be required by the CEO Mr. Raman,to reconcile the differences in approach, interest or opinion in the organisation.

Q5. List any three tasks that Mr. Armstrong needs to do, as a production manager, in his firm, to carry out the plans laid down by the top managers.

Q.6 In a company, the marketing department's objective is to increase sales by 10 per cent by offering discounts. But, the finance department does not approve of such discounts as it means loss of revenue.

These kinds of conflict arise in organisations because of the lack of one of the concepts of management

- (a) Identify and explain the concept of management highlighted above.
- (b) State the characteristics of management the company is violating.